

# Topics and Equations for AP Physics C: Electricity & Magnetism

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**You are permitted use of a scientific calculator and the College Board's AP Physics equation sheet for the ENTIRE exam** (see [www.collegeboard.org/ap/calculators](http://www.collegeboard.org/ap/calculators) for a list of acceptable calculators). The reason for this is explained in the AP Physics Course Description:

*The purpose of allowing calculators and equation sheets to be used in both sections of the exam is to place greater emphasis on the understanding and application of fundamental physical principles and concepts. For solving problems and writing essays, a sophisticated scientific or graphing calculator, or the availability of equations, is no substitute for a thorough grasp of the physics involved. The availability of these equations to all students means that in the scoring of the exam, little or no credit will be awarded for simply writing down equations or for answers unsupported by explanations or logical development.*

The AP Physics equation sheet provides a list of fundamental equations from which all others may be derived. The following list, adapted from Tipler & Mosca's Physics for Scientists and Engineers, Fourth Edition, describes the topics and *all* of the equations with which you should be familiar before you take the AP Physics C: Electricity and Magnetism Exam. **The fundamental equations provided by the College Board are indicated with an asterisk \* on the following pages.**

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## ELECTRIC FIELD: DISCRETE CHARGE DISTRIBUTIONS

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1. Quantization and conservation are fundamental properties of electric charge.
2. Coulomb's law is the fundamental law of interaction between charges at rest.
3. The electric field describes the condition in space set up by a charge distribution.

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### TOPIC

### RELEVANT EQUATIONS AND REMARKS

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#### 1. Charge

There are two kinds of charge, positive and negative.

Charges of like sign repel, those of opposite sign attract.

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##### Quantization

Charge is quantized—it always occurs in integer multiples of the fundamental charge unit  $e$ . The charge of the electron is  $-e$  and that of the proton is  $+e$ .

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##### Magnitude

$$e = 1.60 \times 10^{-19} \text{ C}$$

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##### Conservation

Charge is conserved. When charged particles are created or annihilated, the total amount of charge carried by the created or annihilated particles is zero.

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#### 2. Conductors and Insulators

In metals, about one electron per atom is delocalized (free to move about the entire material). In insulators, all the electrons are bound to nearby atoms.

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##### Ground

A very large conductor (such as Earth) that can supply or absorb a virtually unlimited amount of charge is called a ground.

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#### 3. \* Coulomb's Law

The force exerted by point charge  $q_1$  on point charge  $q_2$  a distance  $r_{12}$  away is given by

$$\vec{F}_{12} = \frac{k q_1 q_2}{r_{12}^2} \hat{r}_{12}$$

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##### Coulomb's constant

$$k = 8.99 \times 10^{-19} \text{ N}\cdot\text{m}^2/\text{C}^2 \approx 9.00 \times 10^{-19} \text{ N}\cdot\text{m}^2/\text{C}^2$$

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## ELECTRIC FIELD: DISCRETE CHARGE DISTRIBUTIONS

TOPIC	RELEVANT EQUATIONS AND REMARKS
<b>4. * Electric Field</b>	The electric field due to a system of charges at a point is defined as the net force $\vec{F}$ exerted by those charges on a very small positive test charge $q_0$ divided by $q_0$ : $\vec{E} \equiv \frac{\vec{F}}{q_0}$
Due to a point charge	$\vec{E} = \frac{kQ}{r^2} \hat{r}$
Due to a system of point charges	The electric field at a point due to several charges is the vector sum of the fields at that point due to the individual charges: $\vec{E}_{net} = \sum_{i=1}^n \vec{E}_i$
<b>5 Electric Field Lines</b>	The electric field can be represented by electric field lines that emanate from positive charges and terminate on negative charges (the direction of the electric field is defined to be the direction a small positive charge would move if placed in the field). The strength of the electric field is indicated by the density of the electric field lines.

## ELECTRIC FIELD: CONTINUOUS CHARGE DISTRIBUTIONS

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1. Gauss's law is a fundamental law of physics that is equivalent to Coulomb's law for static charges.
2. For highly symmetric charge distributions, Gauss's law can be used to calculate the electric field.

### TOPIC

### RELEVANT EQUATIONS AND REMARKS

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#### 1. Electric Field for a Continuous

$$\vec{E} = \int d\vec{E} = \int \frac{k dq}{r^2} \hat{r}$$

##### Charge Distribution

where

$dq = \rho dV$  for a charge distributed throughout a volume

$dq = \sigma dA$  for a charge distributed on a surface

$dq = \lambda dL$  for a charge distributed along a line

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#### 2. Electric Flux

$$\Phi_E = EA \cos \theta = \int \vec{E} \cdot d\vec{A}$$

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#### 3. \* Gauss's Law

$$\Phi_{E, net} = \oint_{surface} \vec{E} \cdot d\vec{A} = \frac{Q_{inside}}{\epsilon_0}$$

The net electric flux through a closed surface equals the net charge within the surface divided by  $\epsilon_0$ .

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Permittivity of free space

$$\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2/\text{N}\cdot\text{m}^2$$

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Coulomb's constant

$$k = \frac{1}{4\pi\epsilon_0}$$

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#### 4. Charge on a Conductor

In electrostatic equilibrium, the charge density is zero throughout the material of the conductor. All excess or deficit charge resides on the outer surfaces of the conductor.

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#### 5. $\vec{E}$ just Outside a Conductor

The resultant electric field just outside the surface of a conductor is normal to the surface and has the magnitude

$$E_n = \frac{\sigma}{\epsilon_0}$$

where  $\sigma$  is the local surface charge density on the conductor.

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## ELECTRIC POTENTIAL

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1. Electric potential at a location, which is defined as the electric potential energy per unit charge that a test charge would have at that location, is an important derived physical concept that is related to the electric field.
2. Because potential is a scalar quantity, it is often easier to calculate than the electric field. Once  $V$  is known,  $\vec{E}$  can be calculated from  $V$ .

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### TOPIC

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### RELEVANT EQUATIONS AND REMARKS

#### 1. Potential Difference

The potential difference  $V_b - V_a$  is defined as the negative of the work per unit charge done by the electric field on a test charge as it moves from point  $a$  to point  $b$ :

$$\Delta V \equiv V_b - V_a = \frac{\Delta U}{q_0} = -\int_a^b \vec{E} \cdot d\vec{s}$$

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Potential from infinitesimal displacements

$$dV = -\vec{E} \cdot d\vec{s}$$

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#### 2. Electric Potential

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Potential due to a point charge

$$V = \frac{kq}{r} = \frac{1}{4\pi\epsilon_0} \frac{q}{r} \quad (V = 0 \text{ if } r = \infty)$$

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\* Potential due to a system of charges

$$V = k \sum_{i=1}^n \frac{q_i}{r_i}$$

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Potential due to a continuous charge distribution

$$V = k \int \frac{dq}{r}$$

where  $dq$  is an increment of charge and  $r$  is the distance from the increment to the field point. This expression can be used only if the charge distribution is contained in a finite volume so that the potential can be chosen to be zero at infinity.

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## ELECTRIC POTENTIAL

TOPIC	RELEVANT EQUATIONS AND REMARKS
Continuity of electric potential	The potential function $V$ is continuous everywhere in space.
<b>3. Computing the Electric Field from the Potential</b>	The electric field points in the direction of the most rapid decrease in the potential.
Gradient	A vector that points in the direction of the greatest rate of change in a scalar function and that has a magnitude equal to the derivative of that function, with respect to the distance in that direction, is called the gradient of the function. $\vec{E}$ is the negative gradient of $V$ .
Potential as a function of $x$ alone	$E_x = -\frac{d}{dx}V(x)$
* Potential as a function of $r$ alone	$E_r = -\frac{d}{dr}V(r)$
<b>4. General Relation between <math>\vec{E}</math> and <math>V</math></b>	$\vec{E} = -\vec{\nabla}V = -\left(\frac{\partial V}{\partial x}\hat{i} + \frac{\partial V}{\partial y}\hat{j} + \frac{\partial V}{\partial z}\hat{k}\right)$
<b>5. Units</b>	
$V$ and $\Delta V$	The SI unit of potential and potential difference is the volt: $1 \text{ V} = 1 \text{ J/C}$
Electric field	$1 \text{ N/C} = 1 \text{ V/m}$
Electron volt	The electron volt (eV) is the change in potential energy of a particle of charge $e$ as it moves from $a$ to $b$ where $V_b - V_a = 1$ volt: $1 \text{ eV} = 1.60 \times 10^{-19} \text{ C} \cdot \text{V} = 1.60 \times 10^{-19} \text{ J}$
<b>6. * Potential Energy of Two Charges</b>	$U = q_0 V = \frac{k q_0 q}{r} \quad (U = 0 \text{ if } r = \infty)$

## ELECTRIC POTENTIAL

TOPIC	RELEVANT EQUATIONS AND REMARKS
<b>7. Charge on a Nonspherical Conductor</b>	On a conductor of arbitrary shape, the surface charge density $\sigma$ is greatest at points where the radius of curvature is smallest.
<b>8. Dielectric Breakdown</b>	The amount of charge that can be placed on a conductor is limited by the fact that molecules of the surrounding medium undergo dielectric breakdown at very high electric fields, causing the medium to become a conductor.
Dielectric strength	The dielectric strength is the magnitude of the electric field at which dielectric breakdown occurs. The dielectric strength of dry air is $E_{max} \approx 3 \times 10^6 \text{ V/m} = 3 \text{ MV/m}$
<b>9. Electrostatic Potential Energy</b>	The electrostatic potential energy of a system of point charges is the work needed to bring the charges from an infinite separation to their final positions.
Of point charges	$U = \frac{1}{2} \sum_{i=1}^n q_i V_i$
Of a conductor with charge $Q$ and potential $V$	$U = \frac{1}{2} QV$
Of a system of conductors	$U = \frac{1}{2} \sum_{i=1}^n Q_i V_i$

## CAPACITANCE

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1. Capacitance is an important defined quantity that relates charge to potential difference.
2. Two devices connected in *parallel* share a common potential difference across each device *due solely to the way they are connected*.
3. Two devices connected in *series* are connected by a conducting path *that contains no junctions*.
4. The changes in potential around any closed path *always* sum to zero. This is known as Kirchhoff's loop rule.

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### TOPIC

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### RELEVANT EQUATIONS AND REMARKS

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#### 1. Capacitor

A capacitor is a device for storing charge and energy. It consists of two conductors that are insulated from each other and carry equal and opposite charges.

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#### 2. \* Capacitance

$$C \equiv \frac{Q}{V}$$

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Single conductor

$Q$  is the conductor's total charge,  $V$  is the conductor's potential relative to its surroundings.

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Capacitor

$Q$  is the magnitude of the charge on either conductor,  $V$  is the magnitude of the potential difference between the conductors.

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Of an isolated spherical conductor

$$C = 4\pi\epsilon_0 R$$

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\* Of a parallel-plate capacitor

$$C = \frac{\epsilon_0 A}{d}$$

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\* Energy stored in a capacitor

$$U = \frac{1}{2}QV = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}CV^2$$

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## CAPACITANCE

### TOPIC

### RELEVANT EQUATIONS AND REMARKS

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#### 3. Equivalent Capacitance

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\* Parallel capacitors

When devices are connected in parallel, the voltage drop is the same across each.

$$C_{eq} = C_1 + C_2 + C_3 + \dots = \sum_{i=1}^n C_i$$

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\* Series capacitors

When capacitors are in series, the voltage drops add. If the total charge on each connected pair of plates is zero, then:

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \dots = \sum_{i=1}^n \frac{1}{C_i}$$

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#### 4. Dielectrics

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Macroscopic behavior

A nonconducting material is called a dielectric. When a dielectric is inserted between the plates of a charged, electrically isolated capacitor, the electric field between the plates is weakened and the capacitance is thereby increased by the factor  $\kappa$ , which is the dielectric constant.

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Microscopic view

The electric field in the dielectric of a capacitor is weakened because the molecular dipole moments (either preexisting or induced) tend to align with the applied field and thereby produce a second electric field inside the dielectric that opposes the applied field. The aligned dipole moment of the dielectric is proportional to the applied field.

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## CAPACITANCE

TOPIC	RELEVANT EQUATIONS AND REMARKS
Electric field inside	$E = \frac{E_0}{\kappa}$
* Effect on capacitance	$C = \kappa C_0$
Permittivity $\epsilon$	$\epsilon = \kappa \epsilon_0$
Uses of a dielectric	<ol style="list-style-type: none"><li>1. Increases capacitance</li><li>2. Increases dielectric strength</li><li>3. Physically separates conductors</li></ol>

## ELECTRIC CURRENT AND DIRECT-CURRENT CIRCUITS

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1. Ohm's law is an empirical law that holds only for certain materials.
2. Current, resistance, and emf are important *defined* quantities.
3. Kirchhoff's rules follow from the conservation of charge and the conservative nature of the electric field.

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### TOPIC

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### RELEVANT EQUATIONS AND REMARKS

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#### 1. \* Electric Current

Electric current is the rate of flow of electric charge through a cross-sectional area.

$$I \equiv \lim_{\Delta t \rightarrow 0} \frac{\Delta Q}{\Delta t} = \frac{dQ}{dt}$$

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#### \* Drift velocity

In a conducting wire, electric current is the result of the slow drift of charge carriers that are accelerated by an electric field in the wire and then collide with the lattice ions. Typical drift speeds of electrons in wires are of the order of a few millimeters per second. For mobile charges moving in the positive direction,

$$I = nqv_dA$$

where  $q$  is the charge per carrier,  $n$  is the charge carrier volume density,  $v_d$  is the drift velocity, and  $A$  is the cross-sectional area of the wire.

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#### Current Density

The current density  $\vec{J}$  is related to the drift velocity by

$$\vec{J} = nq\vec{v}_d$$

The current  $I$  through a cross-sectional surface is the flux of the current density through the surface.

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#### \* $\vec{E}$ in terms of $\vec{J}$

$$\vec{E} = \rho\vec{J}$$

where  $\rho$  is the resistivity of the material.

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## ELECTRIC CURRENT AND DIRECT-CURRENT CIRCUITS

TOPIC	RELEVANT EQUATIONS AND REMARKS
<b>2. Resistance</b>	
Definition of resistance	$R \equiv \frac{V}{I}$
* In terms of resistivity	$R = \frac{\rho L}{A}$
Temperature dependence of resistivity	$\rho = \rho_0[1 + \alpha(T - T_0)]$ where $\alpha$ is the material's temperature coefficient, and $\rho_0$ is the resistivity of the material at the reference temperature $T_0$ , typically 20°C.
<b>3. * Ohm's Law</b>	For ohmic materials, the resistance does not depend on either the current or the potential drop: $V = IR$ ( $R$ constant)
<b>4. Power</b>	
Supplied to a device or segment	$P = IV$
Delivered to a resistor	$P = IV = I^2 R = \frac{V^2}{R}$
<b>5. Emf</b>	
Source of emf	A device that supplies electrical energy to a circuit.
Power supplied by ideal emf source	$P = I\mathcal{E}$

## ELECTRIC CURRENT AND DIRECT-CURRENT CIRCUITS

TOPIC	RELEVANT EQUATIONS AND REMARKS
<b>6. Battery</b>	
Ideal	An ideal battery is a source of emf that maintains a constant potential difference between its two terminals, independent of the current through the battery.
Real	A real battery can be considered as an ideal battery in series with a small resistance, called its internal resistance.
Terminal voltage	The actual potential difference provided between the terminals of a real battery depends on the current through the battery: $\Delta V = V_b - V_a = \mathcal{E} - IR$ where in the battery the positive direction is the direction of increasing potential.
Total energy stored	$E_{\text{stored}} = Q\mathcal{E}$
<b>7. Equivalent Resistance</b>	
Resistors in series	$R_{eq} = R_1 + R_2 + R_3 + \dots = \sum_{i=1}^n R_i$
Resistors in parallel	$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots = \sum_{i=1}^n \frac{1}{R_i}$
<b>8. Kirchhoff's Rules</b>	
	<ol style="list-style-type: none"><li>1. When any closed loop is traversed, the algebraic sum of the changes in potential around the loop must equal zero.</li><li>2. At any junction (branch point) in a circuit where the current can divide, the sum of the currents into the junction must equal the sum of the currents out of the junction.</li></ol>

## ELECTRIC CURRENT AND DIRECT-CURRENT CIRCUITS

TOPIC	RELEVANT EQUATIONS AND REMARKS
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### 9. Discharging a Capacitor

Charge on the capacitor

$$q(t) = Q_0 e^{-t/RC} = Q_0 e^{-t/\tau}$$

Current in the circuit

$$I(t) = \frac{dq}{dt} = \frac{V_0}{R} e^{-t/RC} = I_0 e^{-t/\tau}$$

Time constant

$$\tau = RC$$

### 10. Charging a Capacitor

Charge on the capacitor

$$q(t) = C\mathcal{E}[1 - e^{-t/RC}] = Q(1 - e^{-t/\tau})$$

Current in the circuit

$$I(t) = \frac{\mathcal{E}}{R} e^{-t/RC} = I_0 e^{-t/\tau}$$

## THE MAGNETIC FIELD

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1. The magnetic field describes the condition in space in which moving charges experience a force perpendicular to their velocity.
2. The magnetic force is part of the electromagnetic interaction, one of the three known fundamental interactions in nature.
3. The magnitude and direction of a magnetic field  $\vec{B}$  are defined by the formula  $\vec{F} = q\vec{v} \times \vec{B}$ , where  $\vec{F}$  is the force exerted on a particle with charge  $q$  moving with velocity  $\vec{v}$ .

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### TOPIC

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### RELEVANT EQUATIONS AND REMARKS

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#### 1. Magnetic Force

\* On a moving charge

$$\vec{F} = q\vec{v} \times \vec{B},$$

\* On a current element

$$d\vec{F} = I d\vec{s} \times \vec{B}$$

Between two parallel wires

$$\frac{F}{\ell} = \frac{\mu_0 I_1 I_2}{2\pi r}$$

This force is attractive if the currents are in the same direction and repulsive if they are in opposite directions.

Unit of the magnetic field

The SI unit of magnetic fields is the tesla (T):

$$1 \text{ T} = 1 \text{ N/A}\cdot\text{m}$$

A commonly used non-SI unit is the gauss (G), which is related to the tesla through the conversion:

$$1 \text{ T} = 10^4 \text{ G}$$

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## THE MAGNETIC FIELD

TOPIC	RELEVANT EQUATIONS AND REMARKS
<b>2. Motion of Point Charges</b>	A particle of mass $m$ and charge $q$ moving with speed $v$ in a plane perpendicular to a uniform magnetic field moves in a circular orbit. The period and frequency of the circular motion are independent of the radius of the orbit and of the speed of the particle.
Newton's second law	$qvB = m \frac{v^2}{r}$
Cyclotron period	$T = \frac{2\pi m}{qB}$
Cyclotron frequency	$f = \frac{1}{T} = \frac{qB}{2\pi m}$
Velocity selector	A velocity selector consists of crossed electric and magnetic fields so that the electric and magnetic forces balance for a particle moving with speed $v$ . $v = \frac{E}{B}$
Mass spectrometer	The mass-to-charge ratio of an ion of known speed can be determined by measuring the radius of the circular path taken by the ion in a known magnetic field.
<b>3. Current Loops</b>	
Magnetic dipole moment	$\vec{\mu} = NIA \hat{n}$
Torque	$\vec{\tau} = \vec{\mu} \times \vec{B}$
Potential energy of a magnetic dipole	$U = -\vec{\mu} \cdot \vec{B}$
Net force	The net force on a current loop in a <i>uniform</i> magnetic field is zero.

## SOURCES OF THE MAGNETIC FIELD

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1. Magnetic fields arise from moving charges, and therefore from currents.
2. The Biot–Savart law describes the magnetic field produced by a current element.
3. Ampère’s law relates the line integral of the magnetic field along some closed curve to the current that passes through any surface bounded by the curve.

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### TOPIC

### RELEVANT EQUATIONS AND REMARKS

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#### 1. Magnetic Field $\vec{B}$

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\* Biot-Savart law

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I d\vec{s} \times \hat{r}}{r^2}$$

where  $\hat{r}$  is a unit vector that points to the field point  $P$  from the current element  $d\vec{s}$ , and  $\mu_0$  is a constant of proportionality called the permeability of free space:

$$\mu_0 = 4\pi \times 10^{-7} \text{ T}\cdot\text{m/A}$$

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Due to a straight wire segment

$$B = \frac{\mu_0 I}{4\pi r} (\sin \theta_1 - \sin \theta_2)$$

where  $r$  is the perpendicular distance to the wire and  $\theta_1$  and  $\theta_2$  are the angles subtended at the field point by the ends of the wire.

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Due to a long straight wire

Using the above equation with  $\theta_1 = 90^\circ$  and  $\theta_2 = -90^\circ$ ,

$$B = \frac{\mu_0 I}{2\pi r}$$

The direction of  $\vec{B}$  is such that the magnetic field lines encircle the wire in the direction of the fingers of the right hand if the thumb points in the direction of the current.

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\* Inside a long solenoid

$$B = \frac{\mu_0 N I}{L} = \mu_0 n I$$

where  $n$  is the number of turns per unit length.

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## SOURCES OF THE MAGNETIC FIELD

TOPIC	RELEVANT EQUATIONS AND REMARKS
2. Magnetic Field Lines	Magnetic lines neither begin nor end. Either they form closed loops or they continue indefinitely.
3. Gauss's Law for Magnetism	$\Phi_{B,net} = \oint_{surface} \vec{B} \cdot d\vec{A} = 0$
4. Magnetic Poles	Magnetic poles always occur in north–south pairs. Isolated magnetic poles have not been found.
5. * Ampère's Law	$\oint_C \vec{B} \cdot d\vec{s} = \mu_0 I$ <p>where <math>C</math> is any closed path and <math>I</math> is the current enclosed by the path.</p>
Validity of Ampère's law	Ampère's law is valid only if the currents are steady and continuous. It can be used to derive expressions for the magnetic field for situations with a high degree of symmetry, such as a long, straight, current-carrying wire or a long, tightly wound solenoid.

## MAGNETIC INDUCTION

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1. Faraday's law and Lenz's law are fundamental laws of physics.
2. Self-inductance is a property of a circuit element that relates the flux through the element to the current.

### TOPIC

### RELEVANT EQUATIONS AND REMARKS

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#### 1. Magnetic Flux

\* General definition

$$\Phi_B = \int_{\text{surface}} \vec{B} \cdot d\vec{A}$$

Flat surface bounded by

$$\Phi_B = NBA \cos \theta$$

$N$  turns of wire

where  $A$  is the area of the surface bounded by a single turn.

Unit

$$1 \text{ Wb} = 1 \text{ T} \cdot \text{m}^2$$

#### 2. \* Faraday's Law

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}(NBA \cos \theta)$$

General form

$$\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt}$$

#### 3. Lenz's Law

The induced emf and induced current are in such a direction as to oppose, or tend to oppose, the change that produces them.

Alternative statement

When a magnetic flux through a surface changes, the magnetic field due to any induced current produces a flux of its own—through the same surface and opposite in sign to the original change in flux.

#### 4. Inductance

Self inductance

$$L = \frac{\Phi_B}{I}$$

Mutual inductance

$$M = \frac{\Phi_{B21}}{I_1} = \frac{\Phi_{B12}}{I_2}$$

Unit

$$1 \text{ H} = 1 \text{ Wb/A}$$

## MAGNETIC INDUCTION

TOPIC	RELEVANT EQUATIONS AND REMARKS
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### 5. EMF

* Faraday's law	$\mathcal{E} = -\frac{d\Phi_B}{dt}$
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Motional emf	$\mathcal{E} = -B\ell v$
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* Self inductance (back emf)	$\mathcal{E} = -L\frac{dI}{dt}$
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### 6. Magnetic Energy

* Energy stored in an inductor	$U = \frac{1}{2}LI^2$
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### 7. RL Circuits

Potential difference across an inductor	$\Delta V = \mathcal{E} - Ir = -L\frac{dI}{dt} - Ir$
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where  $r$  is the internal resistance of the inductor. For an ideal inductor  $r = 0$ .

Energizing an inductor	In a single-loop circuit consisting of a resistor that has a resistance $R$ , an inductor that has a self-inductance $L$ , and a battery that has an emf $\mathcal{E}$ , the current does not reach its maximum value instantaneously, but rather takes some time to build up. If the current is initially zero, its value at some later time is given by
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$$I(t) = \frac{\mathcal{E}}{R}(1 - e^{-Rt/L}) = I_{max}(1 - e^{-t/\tau})$$

Time constant	$\tau = \frac{L}{R}$
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## MAGNETIC INDUCTION

### TOPIC

### RELEVANT EQUATIONS AND REMARKS

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De-energizing an inductor

In a single-loop circuit consisting of a resistor that has a resistance  $R$ , and an inductor that has a self-inductance  $L$ , the current does not drop to zero through a resistor instantaneously, but rather takes some time to decrease. If the current is initially  $I_0$ , its value at some later time is given by

$$I = I_0 e^{-t/\tau}$$

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## MAXWELL'S EQUATIONS

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Maxwell's equations summarize the fundamental laws of physics that govern electricity and magnetism.

### TOPIC

### RELEVANT EQUATIONS AND REMARKS

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#### 1. Maxwell's Displacement Current

Ampère's law can be generalized to apply to currents that are not steady (and not continuous) if the current  $I$  is replaced by  $I + I_d$ , where  $I_d$  is Maxwell's displacement current:

$$I_d = \epsilon_0 \frac{d\Phi_E}{dt}$$

Generalized form of Ampère's law

$$\oint_{\text{path}} \vec{B} \cdot d\vec{s} = \mu_0 (I + I_d) = \mu_0 I + \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$$

#### 2. Maxwell's Equations

The laws of electricity and magnetism are summarized by Maxwell's equations.

\* Gauss's law

$$\oint_{\text{surface}} \vec{E} \cdot d\vec{A} = \frac{Q_{\text{inside}}}{\epsilon_0}$$

Gauss's law for magnetism

$$\oint_{\text{surface}} \vec{B} \cdot d\vec{A} = 0$$

Faraday's law

$$\oint_{\text{path}} \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt}$$

Ampère-Maxwell law

$$\oint_{\text{path}} \vec{B} \cdot d\vec{s} = \mu_0 I + \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$$

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