Topics and Equations
for
AP Physics C: Mechanics

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You are permitted use of a scientific calculator and the College Board's AP Physics equation sheet for the ENTIRE exam (see www.collegeboard.org/ap/calculators for a list of acceptable calculators). The reason for this is explained in the AP Physics Course Description:

The purpose of allowing calculators and equation sheets to be used in both sections of the exam is to place greater emphasis on the understanding and application of fundamental physical principles and concepts. For solving problems and writing essays, a sophisticated scientific or graphing calculator, or the availability of equations, is no substitute for a thorough grasp of the physics involved. The availability of these equations to all students means that in the scoring of the exam, little or no credit will be awarded for simply writing down equations or for answers unsupported by explanations or logical development.

The AP Physics equation sheet provides a list of fundamental equations from which all others may be derived. The following list, adapted from Tipler & Mosca's Physics for Scientists and Engineers, Fourth Edition, describes the topics and *all* of the equations with which you should be familiar before you take the AP Physics C: Mechanics Exam. **The fundamental equations provided by the College Board are indicated with an asterisk * on the following pages.**

CONTENTS

MEASUREMENT

TOPIC	RELEVANT EQUATIONS AND REMARKS
1. Units	Physical quantities are numbers that are obtained by taking measurements of physical objects. Operational definitions specify operations or procedures that, if followed, define physical quantities. The magnitude of a physical quantity is expressed as a number times a unit.
2. Base Units	The base units in the SI system are the meter (m), the second (s), the kilogram (kg), the kelvin (K), the ampere (A), the mole (mol), and the candela (cd). The unit(s) of every physical quantity can be expressed in terms of these base units.
3. Units in Equations	Units in equations are treated just like any other algebraic quantity.
4. Conversions	Conversion factors, which are always equal to 1, provide a convenient method for converting from one kind of unit to another.
5. Dimensions	The terms of an equation must have the same dimensions.
6. Scientific Notation	For convenience, very small and very large numbers are generally written as a number between 1 and 10 times a power of 10.
7. Exponents	
Multiplication	When multiplying identical bases, the exponents are added.
Division	When dividing identical bases, the exponents are subtracted.
Raising to a power	When a number containing an exponent is itself raised to a power, the exponents are multiplied.

TOPIC	RELEVANT EQUATIONS AND REMARKS
8. Significant Figures	
Multiplications and division	The number of significant figures in the result of multiplication or division is no greater than the least number of significant figures in any of the numbers.
Addition and subtraction	The result of addition or subtraction of two numbers has no significant figures beyond the last decimal place where both of the numbers being added or subtracted have significant figures.
9. Order of Magnitude	A number rounded to the nearest power of 10 is called an order of magnitude. The order of magnitude of a quantity can often be estimated using plausible assumptions and simple calculations.

MOTION IN ONE DIMENSION

Displacement, velocity, and acceleration are important *defined* kinematic quantities.

TOPIC	RELEVANT EQUATIONS AND REMARKS
1. Displacement	$\Delta x \equiv x_f - x_i$
Graphical interpretation	Displacement is the area under the v_x versus t curve.
2. Velocity	
Average velocity	$v_{x,avg} \equiv \dfrac{\Delta x}{\Delta t}$
Instantaneous velocity	$v \equiv \lim\limits_{\Delta t \to 0} \dfrac{\Delta x}{\Delta t} = \dfrac{dx}{dt}$
Graphical interpretation	The instantaneous velocity is the slope of the x versus t curve.
3. Speed	
Average speed	$\text{average speed} \equiv \dfrac{\text{total distance}}{\text{total time}} = \dfrac{s}{t}$
Instantaneous speed	Instantaneous speed is the magnitude of the instantaneous velocity. $\text{speed} = \lvert v_x \rvert$
4. Acceleration	
Average acceleration	$a_{x,avg} \equiv \dfrac{\Delta v_x}{\Delta t}$
Instantaneous acceleration	$a \equiv \lim\limits_{\Delta t \to 0} \dfrac{\Delta v_x}{\Delta t} = \dfrac{dv_x}{dt} = \dfrac{d^2 x}{dt^2}$
Graphical interpretation	The instantaneous acceleration is the slope of the v_x versus t curve.
Acceleration due to gravity	The acceleration of an object near the surface of Earth in free-fall under the influence of gravity alone is directed downward and has magnitude g = 9.91 m/s$^2 \approx$ 10 m/s^2

MOTION IN ONE DIMENSION

TOPIC	RELEVANT EQUATIONS AND REMARKS
5. Kinematic equations for constant acceleration	
* Velocity	$v_{xf} = v_{xi} + a_x t$
Average velocity	$v_{x,avg} = \frac{1}{2}\left(v_{xi} + v_{xf}\right)$
Displacement in terms of $v_{x,avg}$	$\Delta x = x_f - x_i = v_{x,avg} t = \frac{1}{2}\left(v_{xi} + v_{xf}\right) t$
* Displacement as a function of time	$\Delta x = x_f - x_i = v_{xi} t + \frac{1}{2} a_x t^2$
* v_x^2 as a function of Δx	$v_{xf}^2 = v_{xi}^2 + 2 a_x \Delta x$
6. Displacement & velocity as integrals	Displacement is represented graphically as the area under the v_x versus t curve. This area is the integral of v_x over time from some initial time t_i to some final time t_f and is written: $$\Delta x = \lim_{\Delta t \to 0} \sum v_x \Delta t = \int_{t_i}^{t_f} v_x dt$$ Similarly, change in velocity is represented graphically as the area under the a_x versus t curve: $$\Delta v_x = \lim_{\Delta t \to 0} \sum a_x \Delta t = \int_{t_i}^{t_f} a_x dt$$

MOTION IN TWO DIMENSIONS

TOPIC	RELEVANT EQUATIONS AND REMARKS
1. Vectors	
Definition	Vectors are quantities that have both magnitude and direction. Vectors add like displacements.
Components	The component of a vector in a direction in space is the projection of the vector on an axis in that direction. If $\vec{\boldsymbol{A}}$ makes an angle θ with the positive x direction, its x and y components are $A_x = A\cos\theta$ $A_y = A\sin\theta$
Magnitude	$A = \lvert\vec{\boldsymbol{A}}\rvert = \sqrt{A_x^2 + A_y^2}$
Adding vectors graphically	Two vectors may be added graphically by drawing them with the tail of the second arrow at the head of the first arrow. The arrow representing the resultant vector is drawn from the tail of the first vector to the head of the second.
Adding vectors using components	If $\vec{\boldsymbol{C}} = \vec{\boldsymbol{A}} + \vec{\boldsymbol{B}}$ then $C_x = A_x + B_x$ and $C_y = A_y + B_y$
Unit vectors	A vector $\vec{\boldsymbol{A}}$ can be written in terms of unit vectors $\hat{\boldsymbol{i}}$, $\hat{\boldsymbol{j}}$, and $\hat{\boldsymbol{k}}$, which are dimensionless, have unit magnitude, and lie along the x, y, and z axes, respectively: $\vec{\boldsymbol{A}} = A_x\hat{\boldsymbol{i}} + A_y\hat{\boldsymbol{j}} + A_z\hat{\boldsymbol{k}}$

MOTION IN TWO DIMENSIONS

TOPIC	RELEVANT EQUATIONS AND REMARKS
2. Kinematic Vectors	
Position vector	The position vector $\vec{\boldsymbol{r}}$ points from the origin of the coordinate system to the particle.
Instantaneous velocity vector	The velocity vector $\vec{\boldsymbol{v}}$ is the rate of change of the position vector. Its magnitude is the speed, and it points in the direction of motion. $\vec{\boldsymbol{v}} \equiv \lim_{\Delta t \to 0} \frac{\Delta \vec{\boldsymbol{r}}}{\Delta t} = \frac{d\vec{\boldsymbol{r}}}{dt}$
Instantaneous acceleration vector	$\vec{\boldsymbol{a}} \equiv \lim_{\Delta t \to 0} \frac{\Delta \vec{\boldsymbol{v}}}{\Delta t} = \frac{d\vec{\boldsymbol{v}}}{dt}$
3. Relative Velocity	If a particle *p* moves with velocity $\vec{\boldsymbol{v}}_{pA}$ relative to reference frame *A*, which is in turn moving with velocity $\vec{\boldsymbol{v}}_{AB}$ relative to reference frame *B*, then the velocity of *p* relative to *B* is: $\vec{\boldsymbol{v}}_{pB} = \vec{\boldsymbol{v}}_{pA} + \vec{\boldsymbol{v}}_{AB}$
4. Projectile Motion with No Air Resistance	The +*x* direction is horizontal and the +*y* direction is upward for the equations in this section.
Independence of motion	In projectile motion, the horizontal and vertical motions are independent.
Acceleration	$a_x = 0$ and $a_y = -g$
Dependence on time	$v_x = v_{xi} = v_i \cos\theta$ $v_y = v_i \sin\theta - gt$ $\Delta x = (v_i \cos\theta)t$ $\Delta y = (v_i \sin\theta)t - \frac{1}{2}gt^2$ Alternatively, $\Delta\vec{\boldsymbol{v}} = \vec{\boldsymbol{g}}t$ and $\Delta\vec{\boldsymbol{r}} = \vec{\boldsymbol{v}}_i t + \frac{1}{2}\vec{\boldsymbol{g}}t^2$

MOTION IN TWO DIMENSIONS

TOPIC	RELEVANT EQUATIONS AND REMARKS
Peak projectile height	$H = \frac{(v_i \sin\theta)^2}{2g}$ $H_{max} = \frac{v_i^2}{2g}$ when $\theta = 90^\circ$
Horizontal projectile range	$R = \frac{v_i^2 \sin 2\theta}{g}$ $R_{max} = \frac{v_i^2}{g}$ when $\theta = 45^\circ$
5. Circular Motion	
* Centripetal acceleration	$a_c = \frac{v^2}{r}$
Tangential acceleration	$a_t = \frac{dv}{dt}$
Total acceleration	$a = \sqrt{a_c^2 + a_t^2}$
Period of motion	$T = \frac{\text{distance per revolution}}{\text{speed}} = \frac{2\pi r}{v}$

NEWTON'S LAWS

1. Newton’s laws of motion are fundamental laws of nature that serve as the basis for our understanding of mechanics.
2. Mass is an *intrinsic* property of an object.
3. Force is an important *derived* dynamic quantity.

TOPIC	RELEVANT EQUATIONS AND REMARKS
1. Newton's Laws	
First law	An object at rest stays at rest unless acted on by an external force. An object in motion continues to travel with constant velocity unless acted on by an external force. (Reference frames in which these statement hold are called inertial reference frames.)
* Second law	The acceleration of an object is directly proportional to the net force acting on it. The reciprocal of the mass of the object is the proportionality constant. Thus: $\vec{\boldsymbol{F}}_{net} = \sum \vec{\boldsymbol{F}} = m\vec{\boldsymbol{a}}$
Third law	When two bodies interact, force $\vec{\boldsymbol{F}}_{BA}$ exerted by object B on object A is equal in magnitude and opposite in direction to force $\vec{\boldsymbol{F}}_{AB}$ exerted by object A on object B: $\vec{\boldsymbol{F}}_{BA} = -\vec{\boldsymbol{F}}_{AB}$
2. Inertial Reference Frames	Our statements of Newton’s first and second laws are valid only in inertial reference frames. Any reference frame that is moving with constant velocity relative to an inertial reference frame is itself an inertial reference frame, and any reference frame that is accelerating relative to an inertial frame is not an inertial reference frame. Earth’s surface is, to a good approximation, an inertial reference frame.

TOPIC	RELEVANT EQUATIONS AND REMARKS
3. Force, Mass, and Weight	
Force	Force is defined in terms of the acceleration it produces on a given object. A force of 1 newton (N) is that force which produces an acceleration of 1 m/s^2 on a mass of 1 kg.
Mass	Mass is an intrinsic property of an object. It is the measure of the inertial resistance of the object to acceleration. Mass does not depend on the location of the object. Applying identical forces to each of two objects and measuring their respective accelerations allows the masses of two objects to be compared. The ratio of the masses of the objects is defined to be equal to the inverse ratio of the accelerations produced: $$\frac{m_2}{m_1} \equiv \frac{a_1}{a_2}$$
Gravitational force	The gravitational force $\vec{F}_g$ on an object near the surface of Earth is the force of gravitational attraction exerted by Earth on the object. It is proportional to the gravitational field $\vec{g}$ (which is equal to the free-fall acceleration), and the mass *m* of the object is the proportionality constant: $$\vec{F}_g = m\vec{g}$$ The weight of an object is the magnitude of the gravitational force on the object.

APPLICATIONS OF NEWTON'S LAWS

TOPIC	**RELEVANT EQUATIONS AND REMARKS**
1. * Hooke's Law	When an unstressed spring is compressed or extended by a small displacement $\Delta \vec{x}$, the restoring force it exerts is proportional to $\Delta \vec{x}$: $\vec{F}_x = -k \Delta \vec{x}$
2. Friction	Two objects in contact exert frictional forces on each other. These forces are parallel to the contacting surfaces and directed so as to oppose sliding or tendency to slide.
3. Drag Forces	When an object moves through a fluid, it experiences a drag force that opposes its motion. The drag force increases with increasing speed. If the body is dropped from rest, its speed increases. As it does, the magnitude of the drag force comes closer and closer to the magnitude of the force of gravity, so the net force, and thus the acceleration, approaches zero. As the acceleration approaches zero, the speed approaches a constant value called its terminal speed. The terminal speed depends on both the shape of the body and on the medium through which it falls.

APPLICATIONS OF NEWTON'S LAWS

TOPIC	RELEVANT EQUATIONS AND REMARKS
Drag proportional to velocity	$\vec{\boldsymbol{F}}_D = -b\vec{\boldsymbol{v}}$ Where F_D is the drag force and b is a proportionality constant.
Differential equation for speed	$\frac{dv}{dt} = g - \frac{b}{m}v$
Terminal speed	$v_t = \frac{mg}{b}$
Speed as a function of time	$v = \frac{mg}{b}\left(1-e^{-bt/m}\right) = v_t\left(1-e^{-bt/m}\right)$
Acceleration as a function of time	$a = \frac{dv}{dt} = g\,e^{-bt/m}$
4. Motion Along a Curved Path	A particle moving along an arbitrary curve can be considered to be moving along a circular arc for a short time interval. Its instantaneous acceleration vector has a component $a_c = v^2/r$ toward the center of curvature of the arc and a component $a_t = dv/dt$ that is tangential to the arc. If the particle is moving along a circular path of radius r at constant speed v, $a_t = 0$ and the speed, radius, and period T are related by $2\pi r = vT$.
Centripetal force	$F_c = m a_c = m\frac{v^2}{r}$
Speed around an unbanked curve	$v \leq \sqrt{\mu_s g r}$
Speed at the top of an inside loop	$v \geq \sqrt{g r}$

TOPIC	RELEVANT EQUATIONS AND REMARKS
5. Center of Mass	
* Center of mass for a system of particles	The center of mass of a system of particles is defined to be the point whose coordinates are given by: $x_{cm} = \frac{1}{M}\sum_{i=1}^{n} x_i$ $y_{cm} = \frac{1}{M}\sum_{i=1}^{n} m_i y_i$ $z_{cm} = \frac{1}{M}\sum_{i=1}^{n} z_i$
Center of mass for continuous objects	$\vec{\boldsymbol{r}}_{cm} = \frac{1}{M}\int \vec{\boldsymbol{r}}\, dm$
Position, velocity, and acceleration for the center of mass of a system of particles	$\vec{\boldsymbol{r}}_{cm} = \frac{1}{M}\left(m_1\vec{r}_1+m_2\vec{r}_2+\cdots\right)$ $\vec{\boldsymbol{v}}_{cm} = \frac{1}{M}\left(m_1\vec{\boldsymbol{v}}_1+m_2\vec{\boldsymbol{v}}_2+\cdots\right)$ $\vec{\boldsymbol{a}}_{cm} = \frac{1}{M}\left(m_1\vec{\boldsymbol{a}}_1+m_2\vec{\boldsymbol{a}}_2+\cdots\right)$
Newton's second law for a system	$\sum \vec{\boldsymbol{F}}_{ext} = m\vec{\boldsymbol{a}}_{cm}$

WORK AND KINETIC ENERGY

1. Work, kinetic energy, and power are important *derived* dynamic quantities.
2. The work – kinetic-energy theorem is an important relation derived from Newton's laws applied to a particle. (In this context, a particle is a perfectly rigid object that moves without rotating.)
3. The scalar product of vectors is a mathematical definition that is useful throughout physics.

TOPIC	RELEVANT EQUATIONS AND REMARKS
1. * Work	$W \equiv \int_1^2 \vec{F} \cdot d\vec{s}$
Constant force	$W = \vec{F} \cdot \vec{s} = F s \cos\theta$
Constant force – straight-line motion	$W = F_x \Delta x = F \Delta x \cos\theta$
Variable force – straight-line motion	$W = \int_{x_i}^{x_f} F_x \, dx$ = Area under an F_x versus x curve
2. * Kinetic Energy	$K \equiv \frac{1}{2} m v^2$
3. Work – Kinetic-Energy Theorem	$W = \Delta K = \frac{1}{2} m \left(v_f^2 - v_i^2 \right)$
4. Scalar (Dot) Product	$\vec{A} \cdot \vec{B} \equiv AB \cos\theta$
In terms of components	$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$
Vector times unit vector	$\vec{A} \cdot \hat{i} = A_x$
Derivative product rule	$\frac{d}{dt}\left(\vec{A} \cdot \vec{B} \right) = \frac{d\vec{A}}{dt} \cdot \vec{B} + \vec{A} \cdot \frac{d\vec{B}}{dt}$

WORK AND KINETIC ENERGY

TOPIC	RELEVANT EQUATIONS AND REMARKS
5. * Power	$P = \dfrac{dW}{dt} = \vec{\boldsymbol{F}}\cdot\vec{\boldsymbol{v}}$
6. Center-of-Mass W – KE Theorem	$\int_1^2 \vec{\boldsymbol{F}}_{net}\cdot d\,\vec{\boldsymbol{s}}_{cm} = \Delta K$ This relation is a useful problem-solving tool if for systems that cannot be modeled as a particle.
Center-of-mass work	$W_{cm} = \int_1^2 \vec{\boldsymbol{F}}\cdot d\,\vec{\boldsymbol{s}}_{cm}$
Translational kinetic energy	$K_T = \dfrac{1}{2}Mv^2$, where $M = \sum_{i=1}^{n} m_i$

CONSERVATION OF ENERGY

1. The work – energy theorem and the conservation of energy are fundamental laws of nature that have applications in all areas of physics.
2. The conservation of mechanical energy is an important relation derived from Newton's laws for conservative forces. It is useful in solving many problems.

TOPICS	RELEVANT EQUATIONS AND REMARKS
1. Conservative Force	A force is conservative if the total work it does on a particle is zero when the particle moves along any path that returns it to its initial position. Alternatively, the work done by a conservative force on a particle is independent of the path taken by the particle as it moves from one point to another.
2. Potential Energy	The potential energy of a system is the energy associated with the configuration of the system. The change in the potential energy of a system is defined as the negative of the work done by all internal conservative forces acting on the system.
Definition	$\Delta U \equiv U_2 - U_1 = -W = -\int_1^2 \vec{\boldsymbol{F}} \cdot d\vec{\boldsymbol{s}}$ $dU \equiv -\vec{\boldsymbol{F}} \cdot d\vec{\boldsymbol{s}}$
* Gravitational	$U_g = U_0 + mgy$
* Elastic (spring)	$U_s = \frac{1}{2}kx^2$
Conservative force	$F_x = -\frac{dU}{dx}$

CONSERVATION OF ENERGY

TOPIC	RELEVANT EQUATIONS AND REMARKS
Potential-energy curve	At a minimum on the curve of the potential-energy function versus the displacement, the force is zero and the system is in stable equilibrium. At a maximum, the force is zero and the system is in unstable equilibrium. A conservative force always tends to accelerate a particle toward a position of lower potential energy.
3. Mechanical Energy	The sum of the kinetic and potential energies of a system is defined as the total mechanical energy: $E_{mech} \equiv K_{sys} + U_{sys}$
Work – Energy Theorem for Systems	The total work done on a system by external forces equals the change in mechanical energy of the system less the total work done by internal nonconservative forces: $W_{ext} = \Delta E_{mech} - W_{nc}$
Conservation of Mechanical Energy	If no external forces do work on the system, and if no internal nonconservative forces do work, then the mechanical energy of the system is constant: $K_f + U_f = K_i + U_i$
4. Problem Solving	The conservation of mechanical energy and the work – energy theorem can be used as an alternative to Newton’s laws to solve mechanics problems that require the determination of the speed of a particle as a function of its position.

CONSERVATION OF LINEAR MOMENTUM

The conservation of momentum for an isolated system is a fundamental law of nature that has applications in all areas of physics.

TOPIC	RELEVANT EQUATIONS AND REMARKS
1. Momentum	
* Definition for a particle	$\vec{\boldsymbol{p}} \equiv m\vec{\boldsymbol{v}}$
Kinetic energy of a particle	$K = \frac{p^2}{2m}$
* Newton's second law for a particle	$\vec{\boldsymbol{F}} = \frac{d\vec{\boldsymbol{p}}}{dt}$
Momentum of a system	$\vec{\boldsymbol{p}}_{sys} = \sum_{i=1}^{n} m_i \vec{\boldsymbol{v}}_{\boldsymbol{i}} = M\vec{\boldsymbol{v}}_{sys}$
2. Collisions	
* Impulse	$\vec{\boldsymbol{J}} = \int_{t_i}^{t_f} \vec{\boldsymbol{F}}\,dt = \Delta\vec{\boldsymbol{p}}$
Average force	$\vec{F}_{avg} = \frac{\vec{J}}{\Delta t}$ $\left(\text{so } \vec{J} = \vec{F}_{avg}\Delta t\right)$
Law of conservation of momentum	If the net external force acting on a system remains zero, the total momentum of the system is conserved. For a two particle system: $m_1\vec{\boldsymbol{v}}_{1i} + m_2\vec{\boldsymbol{v}}_{2i} = m_1\vec{\boldsymbol{v}}_{1f} + m_2\vec{\boldsymbol{v}}_{2f}$
Inelastic collisions	Kinetic energy is lost during an inelastic collision.
Perfectly inelastic collisions	Following a perfectly inelastic collision, the two objects stick together and move with a common velocity: $m_1\vec{\boldsymbol{v}}_{1i} + m_2\vec{\boldsymbol{v}}_{2i} = \left(m_1 + m_2\right)\vec{\boldsymbol{v}}_f$
Elastic collisions	An elastic collision between two objects is one in which the sum of their kinetic energies is the same before and after the collision.

ROTATION

1. Angular displacement, angular velocity, and angular acceleration are fundamental defined quantities in rotational kinematics.
2. Torque and moment of inertia are important derived dynamic concepts. Torque is a measure of the effect of a force in changing an object's rate of rotation. Moment of inertia is the measure of an object's inertial resistance to angular accelerations. The moment of inertia depends on the distribution of the mass relative to the rotation axis.
3. The parallel-axis theorem, which follows from the definition of the moment of inertia, often simplifies the calculation of *I*.
4. Newton's second law for rotation, $\sum \vec{\boldsymbol{\tau}} = I\vec{\boldsymbol{\alpha}}$, is derived from Newton's second law and the definitions of τ, I, and α. It is an important relation for problems involving the rotation of a rigid object about an axis of fixed direction.

TOPIC	RELEVANT EQUATIONS AND REMARKS
1. Angular Velocity and Angular Acceleration	
Angular velocity	$\omega \equiv \frac{d\theta}{dt}$
Angular acceleration	$\alpha \equiv \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$
* Tangential speed	$v_t = r\omega$
Tangential acceleration	$a_t = r\alpha$
Centripetal acceleration	$a_c = \frac{v^2}{r} = \omega^2 r$

ROTATION

TOPIC	RELEVANT EQUATIONS AND REMARKS
2. Equations for Rotation with Constant Angular Acceleration	
* Angular velocity	$\omega_f = \omega_i + \alpha t$
Average angular velocity	$\omega_{avg} = \frac{1}{2}(\omega_i + \omega_f)$
* Angular displacement	$\Delta\theta = \theta_f - \theta_i = \omega_i t + \frac{1}{2}\alpha t^2$
ω^2 as a function of $\Delta\theta$	$\omega_f^2 = \omega_i^2 + 2\alpha\Delta\theta$
3. Moment of Inertia	
* System of particles	$I \equiv \sum_{i=1}^{n} m_i r_i^2$
* Continuous object	$I = \int r^2\, dm$
Uniform hoop about center	$I = MR^2$
Uniform solid disk about center	$I = \frac{1}{2}MR^2$
Uniform solid sphere about center	$I = \frac{2}{5}MR^2$
Uniform rod about center	$I = \frac{1}{12}ML^2$
Uniform rod about one end	$I = \frac{1}{3}ML^2$
Parallel-axis theorem	The moment of inertia about an axis a distance *D* from a parallel axis through the center of mass is $I = I_{cm} + MD^2$ where I_{cm} is the moment of inertia about the axis through the center of mass and *M* is the total mass of the object.

ROTATION

TOPIC	RELEVANT EQUATIONS AND REMARKS
4. Energy	
* Rotational kinetic energy	$K_R = \frac{1}{2} I \omega^2$
Power	$P = \tau\omega$
5. * Torque About an Axis	The torque due to a force equals the product of the tangential component of the force and the radial distance from the axis to the point of application of the force: $\tau = Fr\sin\phi = Fd$
6. * Newton's 2nd Law for Rotation	$\tau_{net} = \sum_{i=1}^{n} \tau_i = I\alpha$
7. Nonslip Conditions	If a string that is wrapped around a pulley wheel does not slip, the linear and angular quantities are related by: $v_t = R\omega$ $a_t = R\alpha$
8. Rolling Motion	
Rolling without slipping	$v_{cm} = R\omega$
Total kinetic energy	$K = K_T + K_R = \frac{1}{2} m v_{cm}^2 + \frac{1}{2} I \omega^2$

ANGULAR MOMENTUM

1. Angular momentum is an important derived dynamic quantity in macroscopic physics. In microscopic physics, spin angular momentum is an intrinsic, fundamental property of elementary particles.
2. Conservation of angular momentum is a fundamental law of nature.
3. Quantization of angular momentum is a fundamental law of nature.

TOPIC	RELEVANT EQUATIONS AND REMARKS
1. Vector Nature of Rotation	Right-hand rules are used to obtain the direction of the angular velocity and the torque.
Angular velocity $\vec{\boldsymbol{\omega}}$	The direction of the angular velocity $\vec{\boldsymbol{\omega}}$ is along the axis of rotation in the direction given by the right-hand rule.
Torque $\vec{\boldsymbol{\tau}}$	$\vec{\boldsymbol{\tau}} = \vec{\boldsymbol{r}} \times \vec{\boldsymbol{F}}$
2. Vector (Cross) Product	$\vec{\boldsymbol{A}} \times \vec{\boldsymbol{B}} \equiv AB \sin\phi\, \hat{\boldsymbol{n}}$ where ϕ is the angle between the vectors and $\hat{\boldsymbol{n}}$ is a unit vector perpendicular to the plane of $\vec{\boldsymbol{A}}$ and $\vec{\boldsymbol{B}}$ in the direction given by the right-hand rule as $\vec{\boldsymbol{A}}$ is rotated into $\vec{\boldsymbol{B}}$.
Properties	$\vec{\boldsymbol{A}} \times \vec{\boldsymbol{A}} = 0$ $\vec{\boldsymbol{A}} \times \vec{\boldsymbol{B}} = -\vec{\boldsymbol{B}} \times \vec{\boldsymbol{A}}$ $\frac{d}{dt}(\vec{\boldsymbol{A}} \times \vec{\boldsymbol{B}}) = \left(\vec{\boldsymbol{A}} \times \frac{d\vec{\boldsymbol{B}}}{dt}\right) + \left(\frac{d\vec{\boldsymbol{A}}}{dt} \times \vec{\boldsymbol{B}}\right)$ $\hat{\boldsymbol{i}} \times \hat{\boldsymbol{j}} = \hat{\boldsymbol{k}}$, $\hat{\boldsymbol{j}} \times \hat{\boldsymbol{k}} = \hat{\boldsymbol{i}}$, and $\hat{\boldsymbol{k}} \times \hat{\boldsymbol{i}} = \hat{\boldsymbol{j}}$ $\hat{\boldsymbol{i}} \times \hat{\boldsymbol{i}} = \hat{\boldsymbol{j}} \times \hat{\boldsymbol{j}} = \hat{\boldsymbol{k}} \times \hat{\boldsymbol{k}} = 0$

ANGULAR MOMENTUM

TOPIC	RELEVANT EQUATIONS AND REMARKS
3. Angular Momentum	
* For a point particle	$\vec{\boldsymbol{L}} \equiv \vec{\boldsymbol{r}} \times \vec{\boldsymbol{p}}$ $L = mvr\sin\phi$
* For rotation about an axis	$\vec{\boldsymbol{L}} = I\vec{\boldsymbol{\omega}}$
Newton's 2nd law for angular momentum	$\vec{\boldsymbol{\tau}} = \dfrac{d\vec{\boldsymbol{L}}}{dt}$
Conservation of angular momentum	If the net external torque remains zero, the angular momentum of the system is conserved. (If the component of the net external torque in a given direction remains zero, the component of the angular momentum of the system in that direction remains conserved.)
Rotational kinetic energy	$K_R = \dfrac{L^2}{2I}$

STATIC EQUILIBRIUM

TOPIC	RELEVANT EQUATIONS AND REMARKS
1. Equilibrium of a Rigid Object	
Conditions	1. The net external force acting on the object must be zero: $\sum \vec{\boldsymbol{F}}_{ext} = 0$ 2. The net external torque about any point must be zero: $\sum \vec{\boldsymbol{\tau}}_{ext} = 0$ The sum of the torques about any axis also equals zero.
Stability	The equilibrium of an object can be classified as stable, unstable, or neutral. An object resting on some surface will be in equilibrium if its center of gravity lies over its base of support. Stability can be improved by lowering the center of gravity or by increasing the width of the base.
2. Center of Gravity	The force of gravity exerted on the various parts of an object can be replaced by a single force, the total gravitational force, acting at the center of gravity: $\vec{\boldsymbol{\tau}}_{net} = \sum_i \left(\vec{\boldsymbol{r}}_i \times \vec{\boldsymbol{F}}_{gi}\right) = \vec{\boldsymbol{r}}_{cg} \times \vec{\boldsymbol{F}}_g$ For an object in a uniform gravitational field, the center of gravity coincides with the center of mass.
3. Couples	A pair of equal and opposite forces constitutes a couple. The torque produced by a couple is the same about any point in space. $\vec{\boldsymbol{\tau}} = \left(\vec{\boldsymbol{r}}_2 - \vec{\boldsymbol{r}}_1\right) \times \vec{\boldsymbol{F}}$, so $\tau = FD$ where D is the distance between the lines of action of the forces.

GRAVITATION

1. Kepler's laws are *empirical* observations. They can also be derived from Newton's laws of motion and Newton's law of gravity.
2. Newton's law of gravity is a *fundamental law* of physics, and G is a fundamental *universal* physical constant.
3. The gravitational field is a *fundamental physical concept* that describes the condition in space set up by a mass distribution.

TOPIC	RELEVANT EQUATIONS AND REMARKS
1. Kepler's Three Laws	
Law 1	All of the planets move in elliptical orbits with the Sun at one focus.
Law 2	A line joining any planet to the Sun sweeps out equal areas in equal times.
Law 3	The square of the period of any planet is proportional to the cube of the planet's mean distance from the Sun: $T^2 = Cr^3$ where C has almost the same value for all planets; from Newton's law of gravity, C can be shown to be $4\pi^2/[G(M_S + M_P)]$. If $M_S >> M_P$, this can be expressed as $T^2 = \left(\frac{4\pi^2}{GM_S}\right)r^3$

GRAVITATION

TOPIC	RELEVANT EQUATIONS AND REMARKS
	Kepler's laws can be derived from Newton's law of gravity. The first and third of Kepler's laws follow from the fact that the force exerted by the Sun on the planets varies inversely as the square of the separation distance. The second law follows from the fact that the force exerted by the Sun on a planet is along the line joining them, so the orbital angular momentum of the planet is conserved. Kepler's laws also hold for any object orbiting another in an inverse-square gravitational field, such as a satellite orbiting a planet.
2. * Newton's Law of Gravity	Every point particle exerts an attractive force on every other point particle that is proportional to the masses of the two particles and inversely proportional to the square of the distance separating them: $\vec{\boldsymbol{F}}_{12} = -\frac{G m_1 m_2}{r_{12}^2}\hat{\boldsymbol{r}}_{12}$
* Universal gravitational constant	$G = 6.67\times 10^{-11}\ \mathrm{N{\cdot}m^2/kg^2}$
3. Free-Fall Acceleration	$g = \frac{G M_E}{r^2} = \frac{G M_E}{\left(R_E+h\right)^2}$
4. * Gravitational Potential Energy	The gravitational potential energy U for a system consisting of a particle of mass m outside a spherically symmetric object of mass M and at a distance r from its center is: $U(r) = -\frac{G M m}{r}$ This potential-energy function approaches zero as r approaches infinity.

TOPIC	RELEVANT EQUATIONS AND REMARKS
5. Orbital Mechanical Energy	The mechanical energy E for a system consisting of a particle of mass m outside a spherically symmetric object of mass M and at a distance r from its center is: $E = \frac{1}{2}mv^2 - \frac{GMm}{r}$ However, because gravity provides the centripetal force for a circular orbit, v is dependent on r and $E = -\frac{GMm}{2r}$
Escape Speed	For a given value of r, the speed of the particle for which $E = 0$ is called the escape speed, v_e. On Earth, $v_e = \sqrt{\frac{2GM_E}{R_E}}$
6. Gravitational Field	
Definition	$\vec{\boldsymbol{g}} = \frac{\vec{\boldsymbol{F}}_g}{m}$
Due to Earth	$\vec{g} = -\frac{GM_E}{r^2}\hat{r} \quad (r \geq R_E)$
Due to a thin spherical shell	Outside the shell, the gravitational field is the same as if all the mass of the shell were concentrated at the center. The field inside the shell is zero. $\vec{\boldsymbol{g}} = -\frac{GM}{r^2}\hat{\boldsymbol{r}} \quad (r \geq R)$ $\vec{\boldsymbol{g}} = 0 \quad (r < R)$

OSCILLATIONS

Simple harmonic motion occurs whenever the restoring force is proportional to the displacement from equilibrium.

TOPIC	RELEVANT EQUATIONS AND REMARKS
1. Simple Harmonic Motion	In simple harmonic motion, the acceleration (and thus the net force) is both proportional to, and oppositely directed from, the displacement from the equilibrium position. $F_x = -kx = ma_x$ In general, any motion that satisfies the following differential equation will be simple harmonic motion: $\frac{d^2x}{dt^2} = -\omega^2 x$
Position function	$x = A\cos(\omega t + \phi)$
Velocity function	$v = \frac{dx}{dt} = -\omega A\sin(\omega t + \phi)$
Acceleration function	$a = \frac{dv}{dt} = \frac{d^2x}{dt^2} = -\omega^2 A\cos(\omega t + \phi) = -\omega^2 x$
Maximum speed	$v_{max} = \omega A \ (\text{at } x = 0)$
Maximum acceleration	$a_{max} = \omega^2 A \ (\text{at } x = \pm A)$
Angular frequency	$\omega = 2\pi f = \frac{2\pi}{T}$
Frequency	$f = \frac{1}{T} = \frac{\omega}{2\pi}$
* Period	$T = \frac{1}{f} = \frac{2\pi}{\omega}$
Mechanical energy	$E = K + U = \frac{1}{2}kA^2$
Circular motion	If a particle moves in a circle with constant speed, the projection of the particle onto a diameter of the circle moves in simple harmonic motion.

OSCILLATIONS

TOPIC	RELEVANT EQUATIONS AND REMARKS
General motion near equilibrium	If an object is given a small displacement from a position of stable equilibrium, it typically oscillates about this position with simple harmonic motion.
2. Natural Frequencies for Various Systems	
* Mass on spring	$\omega = \sqrt{\dfrac{k}{m}}$
* Simple pendulum	$\omega = \sqrt{\dfrac{g}{L}}$
Physical pendulum	$\omega = \sqrt{\dfrac{MgD}{I}}$ where I is the moment of inertia about the pivot and D is the distance of the center of mass from the rotation axis.
Torsional oscillator	$\omega = \sqrt{\dfrac{\kappa}{I}}$ where I is the moment of inertia and κ is the torsional constant.