

Roller coasters come in many sizes and styles. They range from simple wooden out-and-back rides to tubular steel behemoths with linear induction motors and magnetic braking systems. Despite differences in design, all coasters are essentially the same after they leave their power source: gravity, and gravity alone, powers the thrills and spills.

You will be designing a roller coaster for this project. This will require you to accurately model the physics involved in coasters in order to make a ride that is both safe and exciting. In order to simplify the task, your coaster will be a traditional coaster where all of the energy for the ride comes from being lifted up a large hill. Your coaster must also have at least one camelback hill and at least one vertical loop. (For simplicity's sake, assume that the loop is perfectly circular; working with the actual shape of the loop, a clothoid curve, greatly increases the difficulty of the project.) Once you've met the requirements you're free to add as many additional ride elements as you wish.



You get to choose the following values:

- height of the lift hill ( $h_i$ )
- speed at the top of the lift hill ( $v_i$ )
- angle of the drop ( $\theta$ )
- number of subsequent hills
- height ( $h$ ) of each subsequent hill
- number of loops
- height ( $h$ ) and radius ( $R$ ) of each loop
- distance ( $d$ ) of each element along the track
- total length ( $d$ ) of track in the ride

You will calculate the following speeds ( $v$ ):

- at the bottom of the first hill
- at the top of each subsequent hill
- at the top of each loop
- at the end of the ride (must be 2 m/s or less)

You will also calculate the cost of your roller coaster.

You must include a description of your engineering firm and an advertisement for your coaster.

- Roller coasters cost \$12,500 per meter and lose speed at the rate of 0.02 m/s per meter.
- The speed  $v_x$  of the coaster at any point  $x$  can be found with the following equation:

$$v_x = \sqrt{v_i^2 + 2g(h_i - h_x)} - (0.02)d_x$$

where  $h_x$  and  $d_x$  are the height of and distance to point  $x$ , respectively. The final term corrects for losses due to friction.

- At the top of a loop, the coaster must meet the following minimum velocity requirement:  $v_{\min} \geq \sqrt{gr_{\text{loop}}}$ . Any speed less than this will cause passengers to fall from the cars. The height of a loop depends on its radius ( $h_{\text{loop}} = 2r_{\text{loop}}$ ) and the length of track required for a loop equals its circumference.
- At the top of a camelback hill, the coaster must meet the following maximum velocity requirement:  $v_{\max} \leq \sqrt{gr_{\text{hill}}}$ . Any speed greater than this will cause passengers to fly out of the cars.