Unit 4: Work, Energy, and Power

There are a number of technical terms in physics that have a nontechnical equivalent in ordinary usage. An example we saw in the previous chapter is force. We can talk about force in conversation without meaning a push or a pull that changes the velocity of an object, but it's easy to see that that technical definition has something in common with the ordinary use of the word *force*. The same is true with *work*, *energy*, and *power*. All three of these words have familiar connotations in ordinary speech, but in physics they take on a technical meaning. As with force, the ordinary meaning of these words provides us with some hint as to their meaning in physics. However, we shouldn't rely too heavily on our intuition, since, as we shall see, there are some significant divergences from what common sense tells us.

The related phenomena of work, energy, and power find their way into a good number of questions in physics. And energy, like force, finds its way into almost every aspect of physics, so a mastery of this subject matter is very important. The conservation of energy is one of the most important laws of physics, and conveniently serves as a tool to sort out many a head-splitting physics problem.

Work

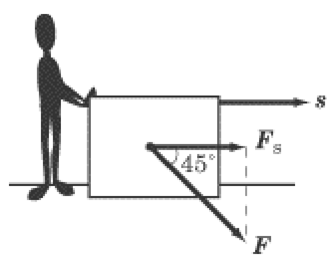
When we are told that a person pushes on an object with a certain force, we only know how hard the person pushes: we don't know what the pushing accomplishes. Work, W, a scalar quantity that measures the product of the force exerted on an object and the resulting displacement of that object, is a measure of what an applied force accomplishes. The harder you push an object, and the farther that object travels, the more work you have done. In general, we say that work is done *by* a force, or *by* the object or person exerting the force, *on* the object on which the force is acting. Most simply, work is the product of force times displacement. However, as you may have remarked, both force and displacement are vector quantities, and so the direction of these vectors comes into play when calculating the work done by a given force. Work is measured in units of joules (J), where $1\text{ J} = 1\text{ N}\cdot\text{m} = 1\text{ kg}\cdot\text{m}^2/\text{s}^2$.

Work When Force and Displacement Are Parallel

When the force exerted on an object is in the same direction as the displacement of the object, calculating work is a simple matter of multiplication. Suppose you exert a force of 10 N on a box in the northward direction, and the box moves 5 m to the north. The work you have done on the box is $W = Fs = (10\text{ N})(5\text{ m}) = 50\text{ N}\cdot\text{m} = 50\text{ J}$. If force and displacement are parallel to one another, then the work done by a force is simply the product of the magnitude of the force and the magnitude of the displacement.

Work When Force and Displacement Are Not Parallel

Unfortunately, matters aren't quite as simple as scalar multiplication when the force and displacement vectors aren't parallel. In such a case, we define work as the product of the displacement of a body and the component of the force in the direction of that displacement. For instance, suppose you push a box with a force *F* along the floor for a distance *s*, but rather than pushing it directly forward, you push on it at a downward angle of 45º. The work you do on the box is not equal to Fs, the magnitude of the force times the magnitude of the displacement. Rather, it is equal to $F_s s$, the magnitude of the force exerted in the direction of the displacement times the magnitude of the displacement.



Some simple trigonometry shows us that $F_s = F\cos\theta$, where θ is the angle between the ***F*** vector and the ***s*** vector. With this in mind, we can express a general formula for the work done by a force, which applies to all cases:

$$W = F_s s = Fs\cos\theta$$

This formula also applies to the cases where ***F*** and ***s*** are parallel, since in those cases $\theta = 0^\circ$, and $\cos 0^\circ = 1$, so $W = Fs$.

Dot Product

What the formula above amounts to is that work is the dot product of the force vector and the displacement vector. As we recall, the dot product of two vectors is the product of the magnitudes of the two vectors multiplied by the cosine of the angle between the two vectors. So the most general vector definition of work is:

$$W = \boldsymbol{F} \cdot \boldsymbol{s} = F s \cos\theta$$

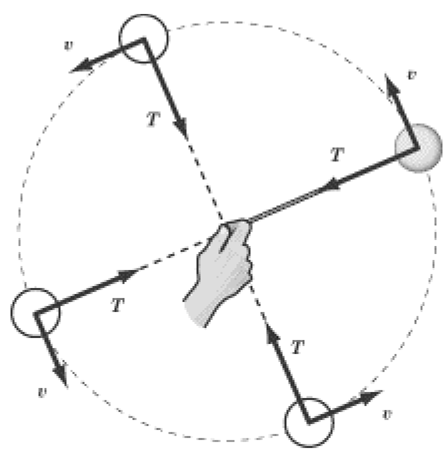
Review

The concept of work is actually quite straightforward, as you’ll see with a little practice. You just need to bear a few simple points in mind:

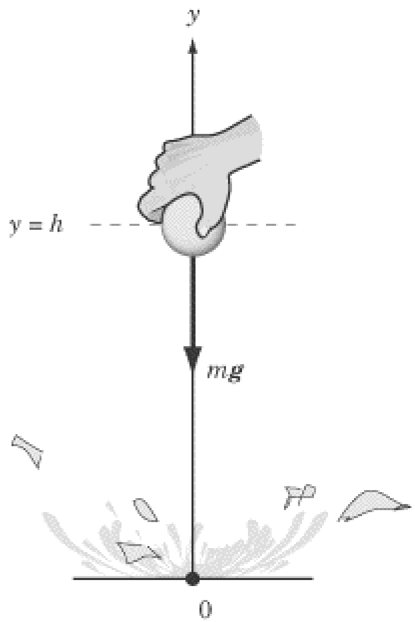
- If force and displacement are both in the same direction, the work done is the product of the magnitudes of force and displacement.
- If force and displacement are at an angle to one another, you need to calculate the component of the force that points in the direction of the displacement, or the component of the displacement that points in the direction of the force. The work done is the product of the one vector and the component of the other vector.
- If force and displacement are perpendicular, no work is done.

Because of the way work is defined in physics, there are a number of cases that go against our everyday intuition. Work is not done whenever a force is exerted, and there are certain cases in which we might think that a great deal of work is being done, but in fact no work is done at all. Let’s look at some examples that you might be encounter:

- You do work on a 10 kg mass when you lift it off the ground, but you do no work to hold the same mass stationary in the air. As you strain to hold the mass in the air, you are actually making sure that it is not displaced. Consequently, the work you do to hold it is zero.
- Displacement is a vector quantity that is not the same thing as distance traveled. For instance, if a weightlifter raises a dumbbell 1 m, then lowers it to its original position, the weightlifter has not done any work on the dumbbell.
- When a force is perpendicular to the direction of an object’s motion, this force does no work on the object. For example, say you swing a tethered ball in a circle overhead, as in the diagram below. The tension force, ***T***, is always perpendicular to the velocity, ***v***, of the ball, and so the rope does no work on the ball.



Example



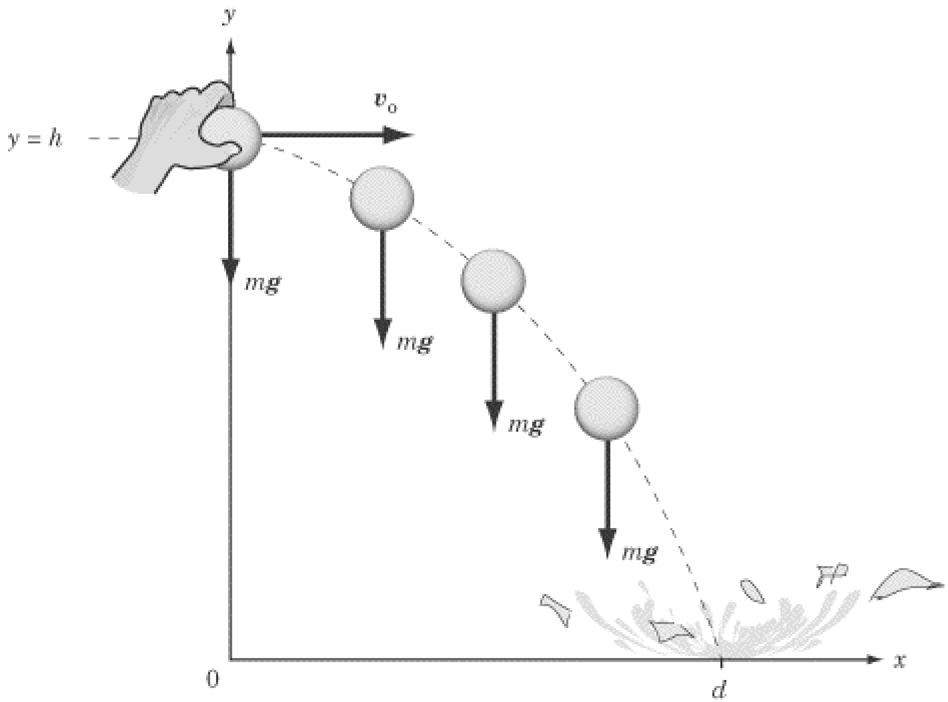
A water balloon of mass *m* is dropped from a height *h*. What is the work done on the balloon by gravity? How much work is done by gravity if the balloon is thrown horizontally from a height *h* with an initial velocity of v_0 ?

What is the work done on the balloon by gravity?

Since the gravitational force of -mg is in the same direction as the water balloon's displacement, -h, the work done by the gravitational force on the ball is the force times the displacement, or $W = (-mg)(-h) = mgh$, *where* $g = 9.8\ m/s^2$.

How much work is done by gravity if the balloon is thrown horizontally from a height h with an initial velocity of v_0***?***

The gravitational force exerted on the balloon is still –mg, but the displacement is different. The balloon has a displacement of –h in the y direction and d (see the figure on the next page) in the x direction. But, as we recall, the work done on the balloon by gravity is not simply the product of the magnitudes of the force and the displacement. We have to multiply the force by the component of the displacement that is parallel to the force. The force is directed downward, and the component of the displacement that is directed downward is –h. As a result, we find that the work done by gravity is mgh, just as before.



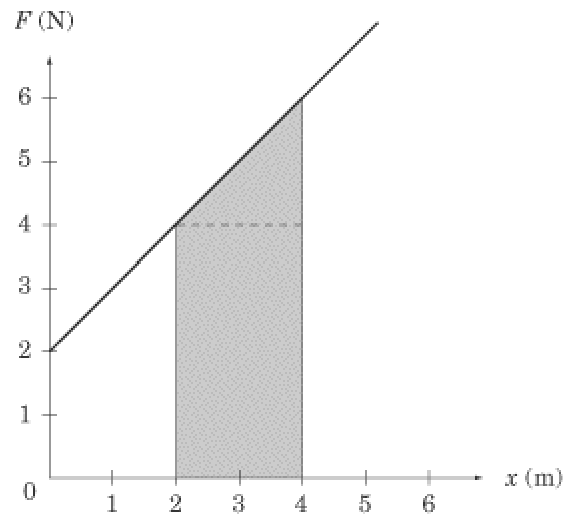
The work done by the force of gravity is the same if the object falls straight down or if it makes a wide parabola and lands 100 m to the east. This is because the force of gravity does no work when an object is transported horizontally, because the force of gravity is perpendicular to the horizontal component of displacement.

Work Problems with Graphs

You may have your understanding of work tested by being asked to interpret a graph. This graph will most likely be a force vs. position graph, though there's a chance it may be a graph of $F\cos\theta$ vs. position. Don't let the appearance of trigonometry scare you: the principle of reading graphs is the same in both cases. In the latter case, you'll be dealing with a graphic representation of a force that isn't acting parallel to the displacement, but the graph will have already taken this into account. Bottom line: all graphs dealing with work will operate according to the same easy principles. The most important thing that you need to remember about these graphs is:

The work done in a force vs. displacement graph is equal to the area between the graph and the *x*-axis during the same interval. If you recall your kinematics graphs, this is exactly what you would do to read velocity on an acceleration vs. time graph, or displacement on a velocity vs. time graph. In fact, whenever you want a quantity that is the product of the quantity measured by the *y*-axis and the quantity measured by the *x*-axis, you can simply calculate the area between the graph and the *x*-axis.

Example



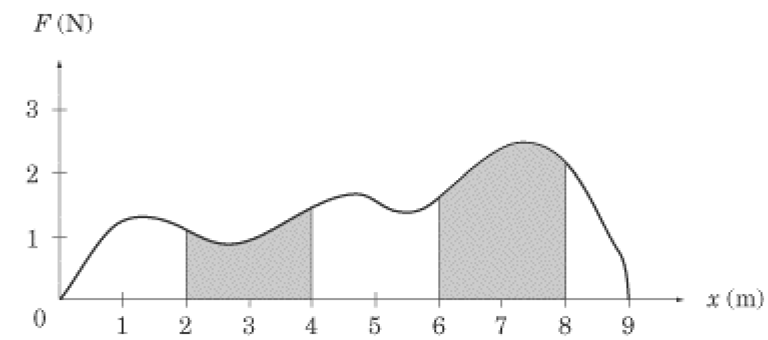
The graph above plots the force exerted on a box against the displacement of the box. What is the work done by the force in moving the box from $x = 2$ m to $x = 4$ m?

The work done on the box is equal to the area of the shaded region in the figure above, or the area of a rectangle of width 2 m and height 4 N plus the area of a right triangle of base 2 m and height 2 N. Determining the amount of work done is simply a matter of calculating the area of the rectangle and the area of the triangle, and adding these two areas together:

$$W = (2\text{ m})(4\text{ N}) + \tfrac{1}{2}(2\text{ m})(2\text{ N}) = 10\text{ J}$$

Curved Force vs. Position Graphs

If a physics test throws you a curved force vs. position graph, don't panic. You won't be asked to calculate the work done, because you can't do that without using calculus. Most likely, you'll be asked to estimate the area beneath the curve for two intervals, and to select the interval in which the most, or least, work was done. In the figure below, more work was done between *x* = 6 m and *x* = 8 m than between *x* = 2 m and *x* = 4 m, because the area between the graph and the *x*-axis is larger for the interval between *x* = 6 m and *x* = 8 m.



Energy

Energy is one of the central concepts of physics, and one of the most difficult to define. One of the reasons we have such a hard time defining it is because it appears in so many different forms. There is the kinetic and potential energy of kinematic motion, the thermal energy of heat reactions, the chemical energy of your iPhone batteries, the mechanical energy of a machine, the elastic energy that helps you launch rubber bands, the electrical energy that keeps most appliances on this planet running, and even mass energy, the strange phenomenon that Einstein discovered and that has been put to such devastating effect in the atomic bomb. This is only a cursory list: energy takes on an even wider variety of forms.

How is it that an electric jolt, a loud noise, and a brick falling to the ground can all be treated using the same concept? Well, one way of defining energy is as a capacity to do work: any object or phenomenon that is capable of doing work contains and expends a certain amount of energy. Because anything that can exert a force or have a force exerted on it can do work, we find energy popping up wherever there are forces.

Energy, like work, is measured in joules (J). In fact, work is a measure of the transfer of energy. However, there are forms of energy that do not involve work. For instance, a box suspended from a string is doing no work, but it has gravitational potential energy that will turn into work as soon as the string is cut. We will look at some of the many forms of energy shortly. First, let's examine the important law of conservation of energy.

Conservation of Energy

As the name suggests, the law of conservation of energy tells us that the energy in the universe is constant. Energy cannot be made or destroyed, only changed from one form to another form. Energy can also be transferred via a force, or as heat. For instance, let's return to the example mentioned earlier of the box hanging by a string. As it hangs motionless, it has gravitational potential energy, a kind of latent energy. When we cut the string, that energy is converted into kinetic energy, or work, as the force of gravity acts to pull the box downward. When the box hits the ground, that kinetic energy does not simply disappear. Rather, it is converted into sound and heat energy: the box makes a loud thud and the impact between the ground and the box generates a bit of heat.

This law applies to any closed system. A closed system is a system where no energy leaves the system and goes into the outside world, and no energy from the outside world enters the system. It is virtually impossible to create a truly closed system on Earth, since energy is almost always dissipated through friction, heat, or sound, but we can create close approximations. Objects sliding over ice or air hockey tables move with a minimal amount of friction, so the energy in these systems remains nearly constant. Problems that quiz you on the conservation of energy will almost always deal with frictionless surfaces, since the law of conservation of energy applies only to closed systems.

The law of conservation of energy is important for a number of reasons, one of the most fundamental being that it is so general: it applies to the whole universe and extends across all time. Because of this, it helps you solve a number of problems that would be very difficult otherwise. For example, you can often determine an object's velocity quite easily by using this law, while it might have been very difficult or even impossible using only kinematic equations. We will see this law at work later in this unit, and again when we discuss elastic and inelastic collisions in the chapter on linear momentum.

Forms of Energy

Though energy is always measured in joules, and though it can always be defined as a capacity to do work, energy manifests itself in a variety of different forms. These various forms pop up all over SAT II Physics, and we will look at some additional forms of energy when we discuss electromagnetism, relativity, and a number of other specialized topics. For now, we will focus on the kinds of energy you'll find in mechanics problems.

Kinetic Energy

Kinetic energy is the energy a body in motion has by virtue of its motion. We define energy as the capacity to do work, and a body in motion is able to use its motion to do work. For instance, a cue ball on a pool table can use its motion to do work on the eight ball. When the cue ball strikes the eight ball, the cue ball comes to a stop and the eight ball starts moving. This occurs because the cue ball's kinetic energy has been transferred to the eight ball.

There are many types of kinetic energy, including vibrational, translational, and rotational. Translational kinetic energy, the main type, is the energy of a particle moving in space and is defined in terms of the particle's mass, m, and velocity, v:

$$K = \tfrac{1}{2} m v^2$$

For instance, a cue ball of mass 0.5 kg moving at a velocity of 2 m/s has a kinetic energy of $K = \tfrac{1}{2}(0.5\text{ kg})(2\text{ m/s})^2 = 1\text{ J}$.

The Work-Kinetic Energy Theorem

If you recall, work is a measure of the transfer of energy. An object that has a certain amount of work done on it has that amount of energy transferred to it. This energy moves the object over a certain distance with a certain force; in other words, it is kinetic energy. This handy little fact is expressed in the work-kinetic energy theorem, which states that the net work done on an object is equal to the object's change in kinetic energy:

$$W = \Delta K$$

For example, say you apply a force to a particle, causing it to accelerate. This force does positive work on the particle and increases its kinetic energy. Conversely, say you apply a force to decelerate a particle. This force does negative work on the particle and decreases its kinetic energy. If you know the forces acting on an object, the work-energy theorem provides a convenient way to calculate the velocity of a particle.

Example

A hockey puck of mass 1.0 kg slides across the ice with an initial velocity of 10 m/s. There is a 1.0 N force of friction acting against the puck. What is the puck's velocity after it has glided 32 m along the ice?

If we know the puck's kinetic energy after it has glided 32 m, we can calculate its velocity. To determine its kinetic energy at that point, we need to know its initial kinetic energy, and how much that kinetic energy changes as the puck glides across the ice.

First, let's determine the initial kinetic energy of the puck. We know the puck's initial mass and initial velocity, so we just need to plug these numbers into the equation for kinetic energy:

$$K = \tfrac{1}{2} m v^2 = \tfrac{1}{2}(1.0\text{ kg})(10\text{ m/s})^2 = 50\text{ J}$$

The friction between the puck and the ice decelerates the puck. The amount of work the ice does on the puck, which is the product of the force of friction and the puck's displacement, is negative.

$$W = \boldsymbol{F} \cdot \boldsymbol{s} = (1.0\text{ N})(32\text{ m})\cos(180^\circ) = -32\text{ J}$$

The work done on the puck by friction decreases its kinetic energy, so after it has glided 32 m the kinetic energy of the puck is 50 - 32 = 18 J.

Now that we know the final kinetic energy of the puck, we can calculate its final velocity by once more plugging numbers into the formula for kinetic energy:

$$K = \tfrac{1}{2} m v^2$$
$$18 \text{ J} = \tfrac{1}{2}(1.0 \text{ kg})\, v^2$$
$$v^2 = 36 \text{ m}^2/\text{s}^2$$
$$v = 6 \text{ m/s}$$

We could also have solved this problem using Newton's Second Law and some kinematics, but the work-energy theorem gives us a quicker route to the same answer.

Potential Energy

As we said before, work is the process of energy transfer. In the example above, the kinetic energy of the puck was transferred into the heat and sound caused by friction. There are a great number of objects, though, that spend most of their time neither doing work nor having work done on them. This book in your hand, for instance, is not doing any work right now, but the second you drop it (whoops!) the force of gravity does some work on it, generating kinetic energy. Now pick up the book and let's continue.

Potential energy, U, is a measure of an object's unrealized potential to have work done on it, and is associated with that object's position in space, or its configuration in relation to other objects. Any work done on an object converts its potential energy into kinetic energy, so the net work done on a given object is equal to the negative change in its potential energy:

$$W = -\Delta U$$

Be very respectful of the minus sign in this equation. It may be tempting to think that the work done on an object increases its potential energy, but the opposite is true. Work converts potential energy into other forms of energy, usually kinetic energy. Remove the minus sign from the equation above, and you are in direct violation of the law of conservation of energy!

There are many forms of potential energy, each of which is associated with a different type of force. Introductory physics usually confines itself to gravitational potential energy and the potential energy of a compressed spring. We will review gravitational potential energy in this section, and the potential energy of a spring in the next unit.

Gravitational Potential Energy

Gravitational potential energy registers the potential for work done on an object by the force of gravity. For example, say that you lift a water balloon to height h above the ground. The work done by the force of gravity as you lift the water balloon is the force of gravity, $-mg$, times the water balloon's displacement, h. So the work done by the force of gravity is $W = -mgh$. Note that there is a negative amount of work done, since the water balloon is being lifted upward, in the opposite direction of the force of gravity.

By making gravity do $-mgh$ joules of work on the water balloon, you have increased its gravitational potential energy by mgh joules (recall the equation $W = -\Delta U$). In other words, you have increased its potential to accelerate downward and cause a huge splash. Because the force of gravity has the potential to do mgh joules of work on the water balloon at height h, we say that the water balloon has mgh joules of gravitational potential energy.

$$U_g = mgh$$

For instance, a 50 kg mass held at a height of 4.0 m from the ground has a gravitational potential energy of:

$$U_g = mgh = (50 \text{ kg})(9.8 \text{ m/s}^2)(4.0 \text{ m}) = 1960 \text{ J}$$

The most important thing to remember is that *the higher an object is off the ground, the greater its gravitational potential energy.*

Mechanical Energy

We now have equations relating work to both kinetic and potential energy:

$$W = \Delta K$$
$$W = -\Delta U$$

Combining these two equations gives us this important result:

$$\Delta K + \Delta U = 0$$

Or, alternatively,

$$\Delta K = -\Delta U$$

As the kinetic energy of a system increases, its potential energy decreases by the same amount, and vice versa. As a result, the sum of the kinetic energy and the potential energy in a system is constant. We define this constant as *E*, the mechanical energy of the system:

$$E = K + U = \text{constant}$$

This law, the conservation of mechanical energy, is one form of the more general law of conservation of energy, and it's a handy tool for solving problems regarding projectiles, pulleys, springs, and inclined planes.

However, mechanical energy is *not* conserved in problems involving frictional forces. When friction is involved, a good deal of the energy in the system is dissipated as heat and sound. The conservation of mechanical energy only applies to closed systems.

Example 1

A student drops an object of mass 10 kg from a height of 5 m. What is the velocity of the object when it hits the ground? Assume, for the purpose of this question, that g = 10 m/s^2.

Before the object is released, it has a certain amount of gravitational potential energy, but no kinetic energy. When it hits the ground, it has no gravitational potential energy, since h = 0, but it has a certain amount of kinetic energy. The mechanical energy, E, of the object remains constant, however. That means that the potential energy of the object before it is released is equal to the kinetic energy of the object when it hits the ground.

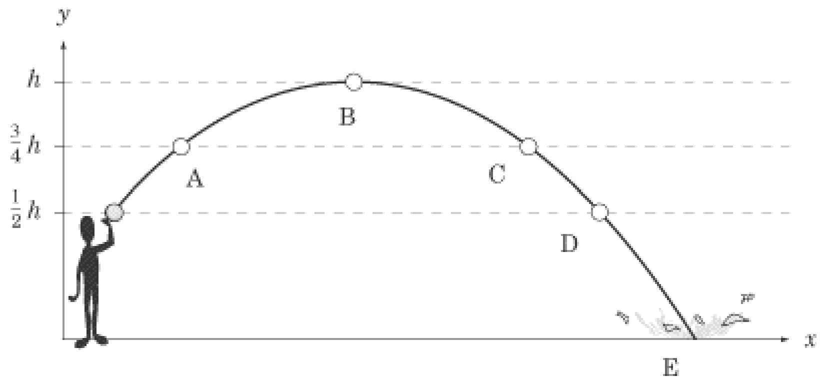
At the moment the object is dropped, it has a gravitational potential energy of:

$$U_g = mgh = (10 \text{ kg})(10 \text{ m/s}^2)(5 \text{ m}) = 500 \text{ J}$$

By the time it hits the ground, all this potential energy will have been converted to kinetic energy. Now we just need to solve for v:

$$K = \tfrac{1}{2} m v^2 = 500 \text{ J}$$
$$\tfrac{1}{2}(10 \text{ kg})\, v^2 = 500 \text{ J}$$
$$v^2 = 100 \text{ m}^2/\text{s}^2$$
$$v = 10 \text{ m/s}$$

Example 2



Consider the above diagram of the trajectory of a thrown tomato:

1. At what point is the potential energy greatest?
2. At what point is the kinetic energy the least?
3. At what point is the kinetic energy greatest?
4. At what point is the kinetic energy decreasing and the potential energy increasing?
5. At what point are the kinetic energy and the potential energy equal to the values at position A?

The answer to question 1 is point B. At the top of the tomato's trajectory, the tomato is the greatest distance above the ground and hence has the greatest potential energy.

The answer to question 2 is point B. At the top of the tomato's trajectory, the tomato has the smallest velocity, since the y-component of the velocity is zero, and hence the least kinetic energy. Additionally, since mechanical energy is conserved in projectile motion, we know that the point where the potential energy is the greatest corresponds to the point where the kinetic energy is smallest.

The answer to question 3 is point E. At the bottom of its trajectory, the tomato has the greatest velocity and thus the greatest kinetic energy.

The answer to question 4 is point A. At this point, the velocity is decreasing in magnitude and the tomato is getting higher in the air. Thus, the kinetic energy is decreasing and the potential energy is increasing.

The answer to question 5 is point C. From our study of kinematics, we know that the speed of a projectile is equal at the same height in the projectile's ascent and descent. Therefore, the tomato has the same kinetic energy at points A and C. Additionally, since the tomato has the same height at these points, its potential energy is the same at points A and C.

Keep this example in mind if you take SAT II Physics, because it is likely that a similar question will appear on the test.

Thermal Energy

There are many cases where the energy in a system seems simply to have disappeared. Usually, this is because that energy has been turned into sound and heat. For instance, a coin sliding across a table slows down and comes to a halt, but in doing so, it produces the sound energy of the coin scraping along the table and the heat energy of friction. Rub your hands together briskly and you will feel that friction causes heat.

We will discuss thermal energy, or heat, in greater detail in a later unit, but it's worth noting here that it is the most common form of energy produced in energy transformations. It's hard to think of an energy transformation where no heat is produced. Take these examples:

- Friction acts everywhere, and friction produces heat.
- Electric energy produces heat: an incandescent light bulb produces far more heat than light.
- When people talk about burning calories, they mean it quite literally: exercise is a way of converting food energy into heat.
- Sounds fade to silence because the sound energy is gradually converted into the heat of the vibrating air molecules. In other words, if you shout very loudly, you make the air around you warmer!

Power

Power is an important physical quantity. Mechanical systems, such as engines, are not limited by the amount of work they can do, but rather by the rate at which they can perform the work. Power, *P*, is defined as the rate at which work is done, or the rate at which energy is transformed. The formula for average power is:

$$P = \frac{W}{\Delta t} = \frac{\Delta E}{\Delta t}$$

Power is measured in units of watts (W), where 1 W = 1 J/s.

Example

A piano mover pushes on a piano with a force of 100 N, moving it 9.0 m in 12 s. With how much power does the piano mover push?

Power is a measure of the amount of work done in a given time period. First we need to calculate how much work the piano mover does, and then we divide that quantity by the amount of time the work takes.

$$W = F s = (100\ \text{N})(9.0\ \text{m}) = 900\ \text{J}$$

$$P = \frac{W}{\Delta t} = \frac{900\ \text{J}}{12\ \text{s}} = 75\ \text{W}$$

Be careful not to confuse the symbol for watts, W, with the symbol for work, W.

Instantaneous Power

Sometimes we may want to know the instantaneous power of an engine or person, the amount of power output by that person at any given instant. In such cases, there is no value for Δt to draw upon. However, when a steady force is applied to an object, the change in the amount of work done on the object is the product of the force and the change in that object's displacement. Bearing this in mind, we can express power in terms of force and velocity:

$$P = \frac{W}{\Delta t} = \frac{F \Delta s}{\Delta t} = F\left(\frac{\Delta s}{\Delta t}\right)$$

$$P = \boldsymbol{F} \cdot \boldsymbol{v}$$

Key Formulas

Work	$W = \boldsymbol{F} \cdot \boldsymbol{s} = F s \cos\theta$
Work Done by Gravity	$W = mgh$
Kinetic Energy	$K = ½ mv^2$
Work-Kinetic Energy Theorem	$W = \Delta K$
Potential Energy	$W = -\Delta U$
Gravitational Potential Energy	$U_g = mgh$
Mechanical Energy	$E = K + U = \text{constant}$
Average Power	$P = \frac{W}{\Delta t} = \frac{\Delta E}{\Delta t}$
Instantaneous Power	$P = \boldsymbol{F} \cdot \boldsymbol{v}$