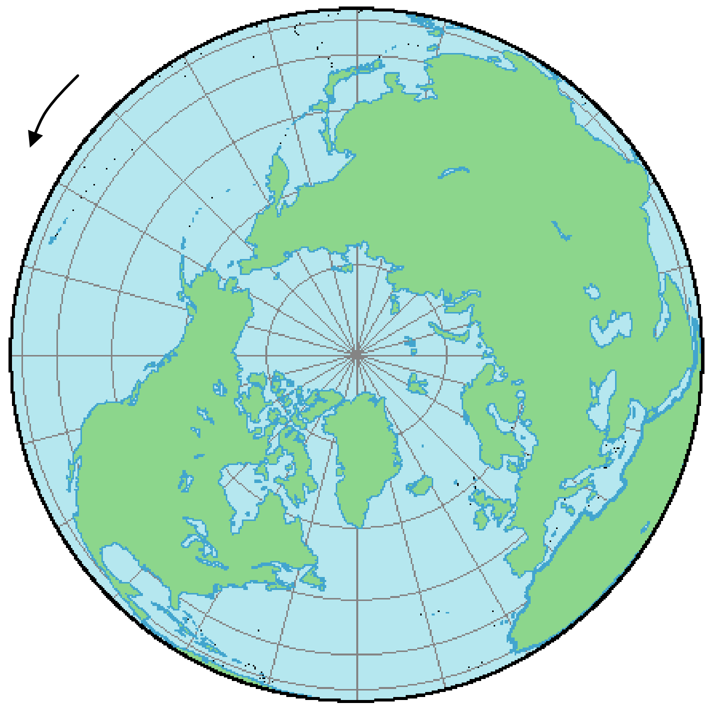
Unit 11: Rotational Kinematics, General Physics

Worksheet 1: Practice working with rotation and revolution



Circular motion can involve rotation and/or revolution. Rotation occurs when the object spins about an internal axis. Revolution occurs when the axis lies outside of the object. Some objects do both at the same time! The time it takes for an object to make one rotation or one revolution is called its period.

Linear motion involves concepts we studied last semester such as displacement (Δx) and velocity ($\Delta x/\Delta t$). Circular motions involve changing an angle ($\Delta\theta$) and angular velocity (ω) which is how much this angle changes with respect to time: ($\Delta\theta/\Delta t$). Additionally, a rotating or revolving object also can move linearly or tangentially. The displacement is an arc around the circumference ($\Delta x = 2\pi r$); the velocity [called tangential velocity (v_t)] is this displacement over time. It is called tangential as the object's velocity is tangential to the arc.

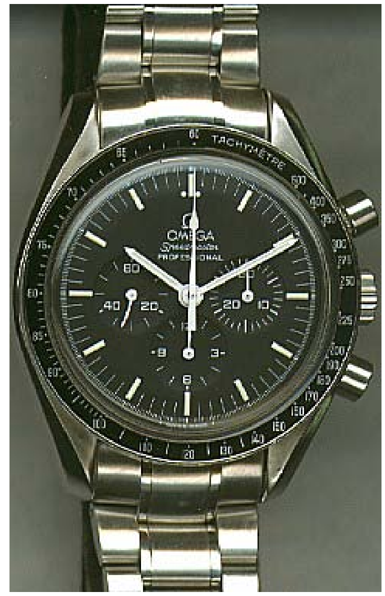
As seen from the North Pole, the earth spins CCW once in 24 hours (actually it is slightly less).

1. (a) What is the $\Delta\theta$ in degrees and radians that the earth moves in 1 hour? (answer = 15 degrees)

(b) What is the Earth's angular velocity (ω) in rpm, degrees per hour, and radians per second? (answers = 6.94×10^{-4} rpm; 15 deg/hr; 7.27×10^{-5} rad/sec)

(c) If the earth's radius is about 6.4×10^{6} meters, what tangential velocity (m/s) does an object have at the equator? (answer = 465.4 m/s)

2. A traditional watch has three hands: an hour hand, a minute hand, and a second hand.



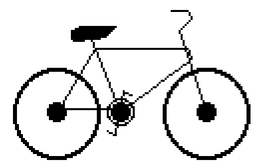
Determine the angular velocity (ω) in radians per second for each hand.

(a) hour hand (answer = 1.45×10^{-4} rad/s):

(b) minute hand (answer = 1.74×10^{-3} rad/s):

(c) second hand ((answer = 1.05×10^{-1} rad/s)

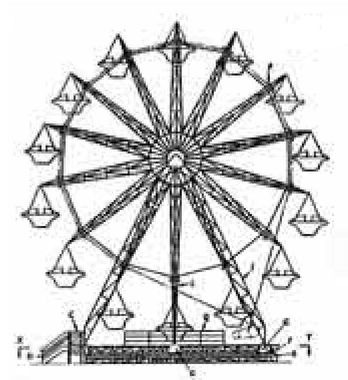
4. A 0.5-meter *diameter* bicycle wheel is rotating at 60 rpm.



(a) What is its initial angular speed in radians per second? (answer = 6.28 or 2π rad/s)

(b) What is the tangential velocity in m/s of a point on the rim of the wheel? (answer = 1.57 m/s)

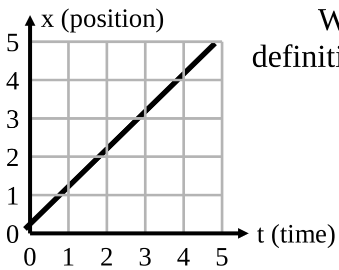
5. Imagine a ferris wheel that is rotating at the rate of 45 degrees each second.
(a) What is the ferris wheel's period of rotation in seconds? (answer = 8 sec)
(b) What is the angular velocity in rotations per minute (rpm)? (answer = 7.5 rpm)
(c) What is the angular velocity in radians per second? (answer = 0.785 rad/s)
(d) If the tangential velocity of one of the cars is 7.85 m/s, how far (in meters) is it located from the center (axis of rotation)? (answer = 10 m)



Unit 11, Worksheet 2: Rotational Kinematics

Last semester we learned that there were two types of linear motion: constant motion (a = 0) and accelerated motion ($a \neq 0$). Accelerated motion involves an object changing its velocity—it might be slowing down (-a) or speeding up (+a).

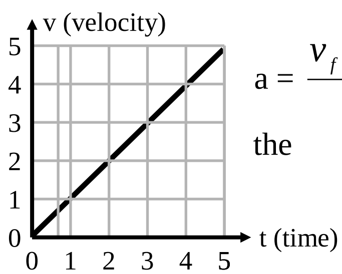
Constant velocity motion: For constant velocity, we defined the velocity of a slope on a position-time graph:



We developed the following equation for constant velocity motion using the definition of slope (the slope could be + or -):

$$v = \frac{x_f - x_i}{t} \quad \text{or} \quad v(t) = x_f - x_i \quad \text{or} \quad \boxed{x_f = x_i + v(t)}$$

Constant accelerated motion: For constant acceleration, acceleration is defined as the slope on a velocity-time graph:



$$a = \frac{v_f - v_i}{t}$$

From this slope equation we were able to come up with the following three basic equations:

$$\boxed{v_f = v_i + at \qquad x = v_i t + \tfrac{1}{2}at^2 \qquad v_f^{\,2} = v_i^{\,2} + 2ax}$$

Why do we review this? Because the same equations can be used for circular motion. Instead of displacement, x, we have an angle, θ; instead of velocity, v, we have angular velocity, ω (radians/sec); instead of acceleration, a, we have angular acceleration, α, (radians/sec^2). The equations are all the same but x, v, and a are replaced by θ, ω, and α.

As with linear motion, we have two types of angular motion: constant angular motion and accelerated angular motion. With constant angular motion, the object changes its θ based on one equation:

$$\boxed{\theta_f = \theta_i + \omega t}$$

If the angular velocity (ω) changes, we have an angular acceleration, α, present. If the angular velocity is increasing (ω is increasing), then α is positive; if the angular velocity is decreasing (ω is decreasing), then α is negative. The three equations we developed for linear acceleration above become the following:

$$\boxed{\omega_f = \omega_i + \alpha t \qquad \theta = \omega_i t + \tfrac{1}{2}\alpha t^2 \qquad \omega_f^{\,2} = \omega_i^{\,2} + 2\alpha\theta}$$

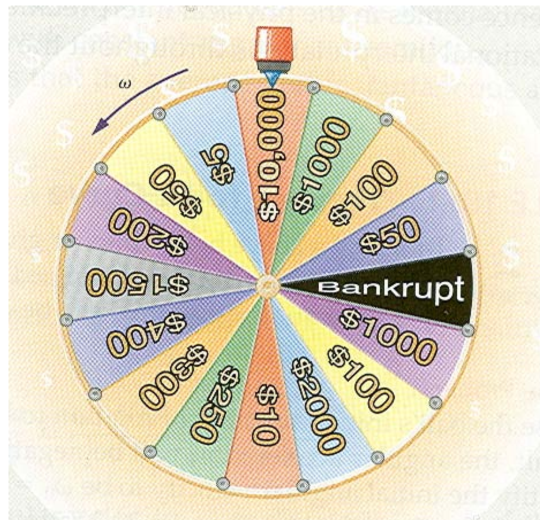
Look at how the equations are very similar to the ones for linear motion above. The only things that are different are the variables! **(One other note: you must use radians for angular quantities)**

Let's see how we would work out a few problems:

Worksheet 2, Rotational Kinematics, continued:
Note: for my calculations using π, I use 3.14159. Your answers may be a bit different depending upon your value for pi and rounding.

Example with constant angular motion:
The second hand of a watch rotates at a constant angular velocity (ω) of 0.105 radians/sec.

a) What is the angular displacement ($\Delta\theta$) in radians and degrees after 90 seconds? (answer = 9.45 radians or 541 degrees)



b) How many rotations did the second hand undergo?
(answer = 1.5 rotations)

Example with constant angular acceleration:
Wheel of Misfortune. In a popular game show, contestants give the wheel a spin and try to win money and prizes! The wheel is given a rotational velocity and this rotation slows down over time and the wheel eventually stops.

One contestant gives the wheel an initial angular velocity (ω_i) of 1 revolution every two seconds. Because of friction, the wheel eventually comes to rest (ω_f = 0 radians/sec) in 6 seconds.

(a) Because the wheel's angular velocity is changing, we must have an angular acceleration. Calculate, α, the angular acceleration in radians/s^2. (Note: you must first change the ω_i of 1 revolution every two seconds into rad/sec). (answer = π rad/sec)

(b) Why is the angular acceleration you found in part (a) a negative value? What does the negative sign physically mean for the wheel? (answer: slowing down)

(c) How many radians ($\Delta\theta$ or $\theta_f - \theta_i$) did the wheel "sweep" out as it was slowing down to a stop? (answer = 9.4 rads)

(d) Convert this $\Delta\theta$ you found in the previous answer from radians to rotations. (answer = 1.5 rotations)

(e) The wheel stop rotating in 6 seconds. So at t = 3 seconds, it is still rotating. Calculate the ω in radians per second at this time. (answer = 1.57 rad/s)

Worksheet 2, Rotational Kinematics, continued:

Note: for my calculations using π, I use 3.14159. Your answers may be a bit different depending upon your value for pi and rounding.

1. A fan that is turning at 10 rpm speeds up to 25 rpm in 10 seconds. How many radians and rotations does the blade require to alter its speed? (Note: You must first convert the rpm to rad/s. To use the equations, any angular quantities must be in radians.) (answer: 18.4 radians or 2.92 rotations)

2. An old phonograph played some records at 45 rpm or 4.71 rad/sec. Let's say the phonograph is turning at 45 rpm and then the motor is turned off, taking 0.75 seconds to come to a stop.

a) What is its average angular acceleration? (answer = - 6.28 rad/sec^2)

b) How many rotations did it make while coming to a stop? (θ = 1.77 radians or 0.281 rot)

3. A flywheel turning at 1200 rev/min (125.7 rad/s) constant angular velocity has a radius of 2.5 cm (0.025 m). As it turns, a string is to be wound onto its rim. How long a piece string will be wrapped in 10 seconds (i.e. how high will it lift the weight)? (answer = 31.4 m)

4. A wheel makes 4.0 rotations in 1 second, rotating at constant angular velocity. What will its angular displacement be after 13.0 s? Determine in rotations and radians. (answers = 52 rotations or 327 rad)

5. CDs are not only used in the music industry, but are also utilized in the computer industry. The information of a huge library can be stored on a single CD. The CD spins around and information is read or written to it as it rotates. One CDROM drive I saw mentions it has a stated angular velocity of 8560 rpm. If the CD starts from rest, what is the angular acceleration if it takes 120 milliseconds (0.12 seconds) for the CD to reach this angular velocity? (answer = 7470 rad/s^2)

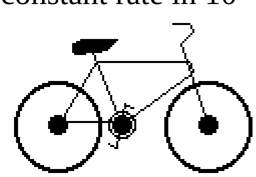
6. Starting from rest, the tub of a washing machine reaches an angular speed of 5.2 rad/s, with an average angular acceleration of 4.0 rad/s^2.

(a) How long does it take the spin cycle to come up to speed? (answer = 1.3 sec)

(b) What angular displacement (in radians and rotations) does the tub rotate through as it reaches this angular velocity? (answer 3.38 radians or 0.54 rotations)

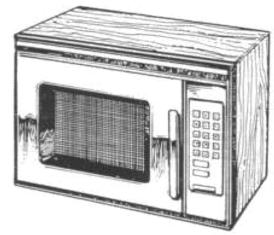
7. The blades of an electric blender are whirling with an angular velocity of +375 rad/s while the "puree" button is pushed in. When the "blend" button is then pressed, the blades accelerate and reach a greater angular velocity after the blades have rotated through an angular displacement of +44.0 rad (seven revolutions). The angular acceleration has a constant value of +1740 rad/s^2. Find the final angular velocity (ω_f) of the blades. (answer = 542 rad/s)

8. A 0.5-meter diameter bicycle wheel initially rotating at 60 rpm rolls to rest at a constant rate in 10 seconds. What is its angular acceleration, α, in radians per second per second? (answer = -0.628 rad/s^2)



Unit 11, Worksheet 3. Several more practice problems dealing with circular motion

Note: for my calculations using π, I use 3.14159. Your answers may be a bit different depending upon your value for pi and rounding.



1. Many microwave ovens rotate the food as it cooks it. Let's say we have a microwave oven with a rotating plate of 15 cm (0.15 cm) radius. The angular acceleration (α) of this rotating plate has been measured at 0.87 rad/s^2. This is the angular acceleration needed to bring the plate from rest to its operational rotational velocity (ω_f). The plate takes 0.5 seconds to reach this ω_f. Once it reaches this ω_f, the plate moves at a constant angular velocity.

(a) What is the angle (θ) the plate moves through in both radians and rotations as it starts from rest and reaches its operational angular velocity? (answer = 0.11 rad or 0.017 rotation)

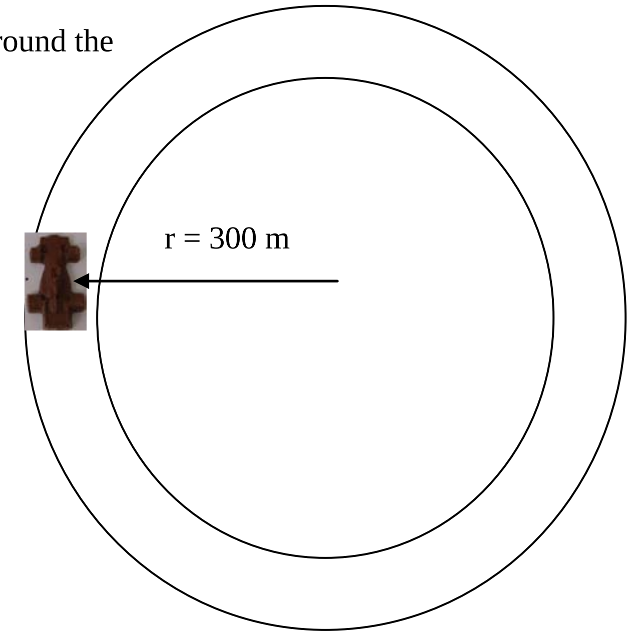
(b) What is the operational angular velocity in radians per second? (answer = 0.44 rad/s).

(c) What would is the tangential velocity (v_t) of a point on the outer edge of the plate? (note: there is an easy way to get the answer and a more difficult way). answer = 0.065 or 0.066m/s).

(d) When the microwave is turned off, the rotating plate makes half of a revolution before stopping. What is the angular acceleration needed to stop the plate given the operational angular velocity of 0.44 rad/s found in part b? (answer = -0.031 rad/s^2)

2. A race car is on a circular track with a radius of 0.30 km (300 m). The driver accelerates from rest with a constant angular acceleration (α) of 4.5 x 10^{-3} rad/s^2. The driver constantly accelerates as he drives one lap around the track.

(a) How long does it take for the driver to make one lap around the track? (answer = 53 seconds)



(b) What is the driver's angular velocity (ω_f) as he finishes this first lap? Answer in rad/s and degrees/sec (answer 0.23 or 0.24 rad/s or around 13-14 degrees per sec)

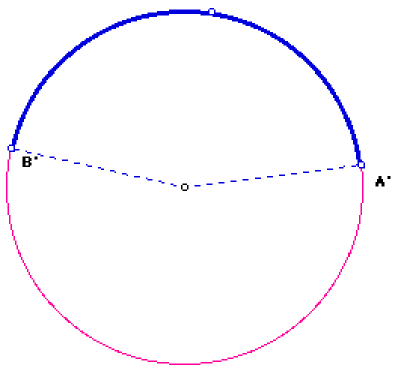
3. The blades of a circular fan running at low speed turn at 250 rpm. When the fan is switched to high speed, the rotation rate increases to 350 rpm. This change of the rotation rate occurs uniformly and takes 5.75 seconds. Remember to convert rpm to rad/s.

(a) What is the angular acceleration (α) needed to go from low to high speed? (answer = 1.82 rad/s^2)

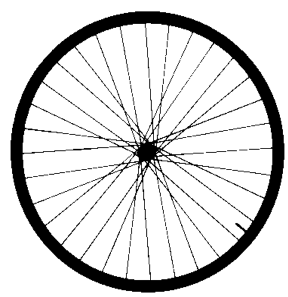
(b) How many rotations do the fan blades go through while the fan is accelerating? (answer = 28.8 rotations)

Physics (worksheet 4, full solutions at the back…)
More practice with rotational questions
Your answers may vary a bit depending upon rounding of pi.

1. While riding on a merry-go-round, a child travels through an arc length of 11.5 m. If the merry-go-round has a radius of 4 m, through what angle (theta) does the child travel? Give the angle in radians, degrees, and rotations. *(answers: 2.875 radians, 164.7 degrees, and 0.46 rotations)*

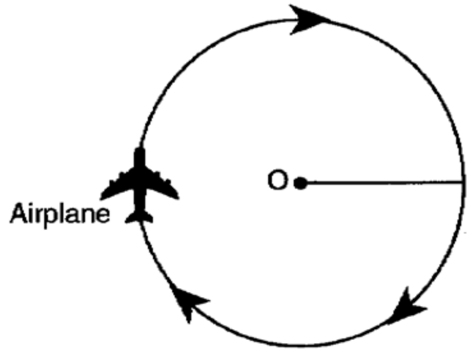


2. A beetle sits stuck in the tread atop a bicycle wheel with a radius of 0.375 m. Assuming the wheel turns counterclockwise, what is the angular displacement of the beetle before it is squashed under the wheel? What arc length does the beetle travel through before it is squashed? *(answers: pi or 3.14 radians, arc length = 1.178 m*)

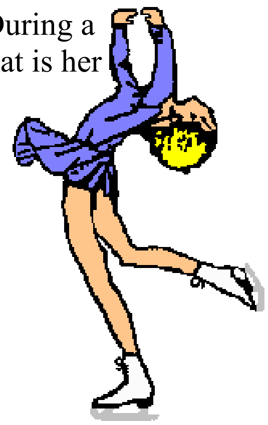


3. A car tire rotates at a constant angular velocity of 3.5 rotations during a time interval of 0.75 s. What is the angular speed of the tire in radians per sec, degrees per sec, and rotations per minute? *(answers = 29.3 radians/sec, 1681 degrees per second, and 280 rotations per minute)*

4. A girl ties a toy airplane to the end of a string and swings it around her head in a horizontal circle with a constant angular velocity of 21 rpm. (a) what is this rotational velocity in radians per second? (b) What would be angular displacement (θ) in radians and degrees if she continued to spin the plane at this constant angular velocity for 10 seconds? *(answers: 2.2 radians/second, 22 radians, 1260 or 1261 degrees)*



5. A figure skater begins spinning counterclockwise at an angular speed of 4.0π rad/s. During a 3.0 sec interval, she slowly pulls her arms inward and finally spins at 8.0π rad/s. (a) What is her average angular acceleration during this time interval? (b) How many rotations did she spin in this time interval as she accelerated her spin? *[answers: a) 4.2 rads/s^2 and b) 9 rotations]*



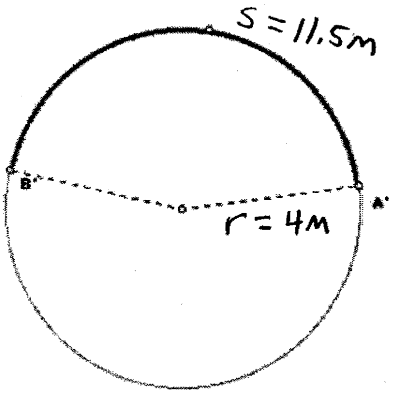
6. You go out on some nice afternoon after school and play with your RC car. You have it moving so that the wheels are moving with an initial angular velocity of 10.8 radians/sec. You accelerate the car with your remote control at a rate of 22.4 radians/sec^2 (this is α). After 3 complete rotations (this is a θ but you'll need to convert this to radians) of the car's wheels you stop the angular acceleration. (a) What is the wheel's final angular velocity in radians per sec? (b) How long (in seconds) did this acceleration take? *(answers: 31 radians/sec and 0.9 seconds)*

My Key

Physics Worksheet 4
More practice with rotational questions (do before the exam)
Your answers may vary a bit depending upon rounding of pi.

1. While riding on a merry-go-round, a child travels through an arc length of 11.5 m. If the merry-go-round has a radius of 4 m, through what angle (theta) does the child travel? Give the θ angle in radians, degrees, and rotations. *(answers: 2.875 radians, 164.7 degrees, and 0.46 rotations)*

First find the θ in radians & then convert this to degrees & rotations



$\theta = \frac{s}{r}$ (when using the radian)

$\theta = \frac{11.5m}{4m} = 2.875$ radians

Put in degrees

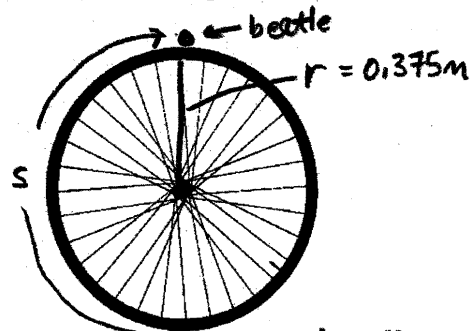
$2.875 \text{ radians} \times \frac{360°}{2\pi \text{ radians}} = 164.7°$

Put in rotation

$2.875 \text{ radians} \times \frac{1 \text{ rotation}}{2\pi \text{ radians}} = 0.46$ rotations

2. A beetle sits stuck in the tread atop a bicycle wheel with a radius of 0.375 m. Assuming the wheel turns counterclockwise, what is the angular displacement of the beetle before it is squashed under the wheel? What arc length does the beetle travel through before it is squashed? *(answers: pi or 3.14 radians, arc length = 1.178 m)*

θ



(a) Well, the wheel has to travel 1/2 rotation from top to bottom. At the bottom, the bug is squashed!

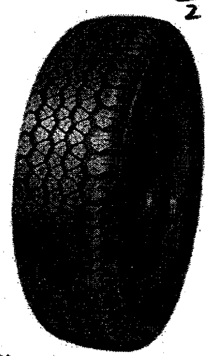
$\frac{1}{2} \text{ rotation} \times \frac{2\pi \text{ radians}}{1 \text{ rotation}} = \pi \text{ radians} = 3.14$ radians

(b) Arc length = S $\theta = \frac{s}{r}$ when θ is put in radians

$\pi \text{ radians} = \frac{s}{0.375}$ $s = 1.178$ meter

or you could take $\frac{1}{2}$ of the wheels circumference $\frac{1}{2}(2\pi R)$ and get the same answer.

3. A car tire rotates at a constant angular velocity of 3.5 rotations during a time interval of 0.75 s. What is the angular speed of the tire in radians per sec, degrees per sec, and rotations per minute? *(answers = 29.3 radians/sec, 1681 degrees per second, and 280 rotations per minute)*



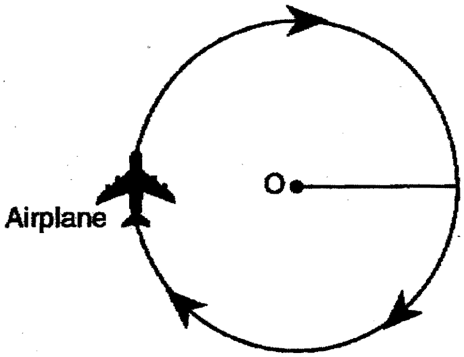
$w = \frac{3.5 \text{ rotations}}{0.75 \text{ sec}} = 4.67 \frac{\text{rot}}{\text{sec}}$

$4.67 \frac{\text{rot}}{\text{sec}} \times \frac{2\pi \text{ radians}}{1 \text{ rot}} = 29.3 \frac{\text{radians}}{\text{sec}}$

$4.67 \frac{\text{rot}}{\text{sec}} \times \frac{360°}{1 \text{ rot}} = 1681$ deg/sec

$4.67 \frac{\text{rot}}{\text{sec}} \times \frac{60 \text{ sec}}{1 \text{ min}} = 280.2 \frac{\text{rotation}}{\text{min}}$ (rpm) my key

4. A girl ties a toy airplane to the end of a string and swings it around her head in a horizontal circle with a constant angular velocity of 21 rpm. (a) what is this rotational velocity in radians per second? (b) What would be angular displacement (θ) in radians and degrees if she continued to spin the plane at this constant angular velocity for 10 seconds? *(answers: 2.2 radians/second, 22 radians, 1260 degrees)*



$\omega = 21 \frac{\text{rotations}}{\text{min}}$

(a) $21 \frac{\text{rotations}}{\text{min}} \times \frac{1\text{ min}}{60\text{ sec}} \times \frac{2\pi \text{ radians}}{1\text{ rotation}}$

$= 2.2 \text{ radians/sec}$

(b) $\theta = \omega t$

or

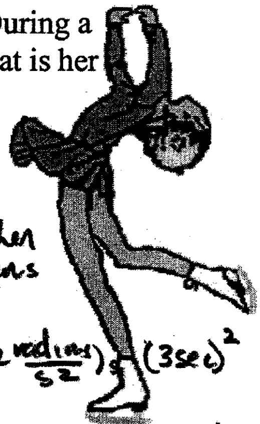
$\Delta\theta = \omega t$

$\Delta\theta = 2.2 \frac{\text{rad}}{\text{sec}} \times 10\text{ sec}$

$= 22 \text{ radians}$

$22 \text{ radians} \times \frac{360°}{2\pi \text{ radians}} = 1260$ or 1261 degrees

5. A figure skater begins spinning counterclockwise at an angular speed of 4.0π rad/s. During a 3.0 sec interval, she slowly pulls her arms inward and finally spins at 8.0π rad/s. (a) What is her average angular acceleration during this time interval? (b) How many rotations did she spin in this time interval as she accelerated her spin? *[answers: a) 4.2 rads/s^2 and b) 9 rotations]*



$\omega_i = 4\pi \text{ rad/sec}$ $\quad \omega_f = 8\pi \text{ rad/sec}$ $\quad t = 3\text{ sec}$

(a) $\omega_f = \omega_i + \alpha t$

$8\pi \frac{\text{radians}}{\text{sec}} = 4\pi \frac{\text{radians}}{\text{sec}} + \alpha(3\text{ sec})$

$\alpha = 4.2 \text{ radians/sec}^2$

(b) Find θ first in radians & then convert to rotations

$\Delta\theta = \omega_i t + \frac{1}{2}\alpha t^2$

$\Delta\theta = 4\pi \frac{\text{radians}}{\text{sec}}(3\text{ sec}) + \frac{1}{2}\left(4.2 \frac{\text{radians}}{\text{s}^2}\right)(3\text{ sec})^2$

$\Delta\theta = 56.599 \text{ rads} \times \frac{1\text{ rot}}{2\pi \text{ radians}} = 9$ rotations

6. You go out on some nice afternoon after school and play with your RC car. You have it moving so that the wheels are moving with an initial angular velocity of 10.8 radians/sec. You accelerate the car with your remote control at a rate of 22.4 radians/sec^2. After 3 complete rotations of the car's wheels you stop the angular acceleration. (a) what is the wheel's final angular velocity in radians per sec? (b) How long (in seconds) did this acceleration take? *(answers: 31 radians/sec and 0.9 seconds)*

$\omega_i = 10.8 \frac{\text{radians}}{\text{sec}}$ $\quad \alpha = 22.4 \frac{\text{radians}}{\text{sec}^2}$ $\quad \theta = 3 \text{ rotations} \times \frac{2\pi \text{ radians}}{\text{sec}} = 18.85 \text{ radians}$

(a) $\omega_f^2 = \omega_i^2 + 2\alpha(\Delta\theta)$

$\omega_f^2 = \left(10.8 \frac{\text{radians}}{\text{sec}}\right)^2 + 2\left(22.4 \frac{\text{radians}}{\text{sec}^2}\right)(18.85 \text{ radians})$

$\omega_f^2 = 116.64 + 844.48$

$\omega_f^2 = 961.12 \text{ rads}^2/\text{sec}^2$

$\omega_f = 31 \text{ radians/sec}$

(31.002)

(b) $\omega_f = \omega_i + \alpha t$

$31 \text{ radians/sec} = 10.8 \frac{\text{radians}}{\text{sec}} + 22.4 \frac{\text{radians}}{\text{sec}^2}(t)$

$20.2 \frac{\text{radians}}{\text{sec}} = 22.4 \frac{\text{radians}}{\text{sec}^2}(t)$

$t = 0.9$ seconds